







TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS

# ADVANCED MATHEMATICS



IEA

TIMSS & PIRLS  
International Study Center  
Lynch School of Education, Boston College





# About TIMSS Advanced 2015

In 2015, IEA and its TIMSS & PIRLS International Study Center at Boston College conducted TIMSS 2015 at fourth and eighth grades and TIMSS Advanced 2015 for students in the final year of secondary school enrolled in advanced mathematics and physics programs or tracks. Both TIMSS 2015 and TIMSS Advanced 2015 provide 20-year trend measures for countries that participated in the first TIMSS assessments in 1995.

TIMSS 2015 and TIMSS Advanced 2015 continue the long history of international assessments in mathematics and science conducted by IEA – the International Association for the Evaluation of Educational Achievement. IEA is an independent international cooperative of national research institutions and government agencies that has been conducting studies of cross-national achievement since 1959. IEA pioneered international comparative assessments of educational achievement in the 1960s to gain a deeper understanding of the effects of policies across countries' different systems of education.

IEA's TIMSS & PIRLS International Study Center is located in the Lynch School of Education at Boston College and has been responsible for directing TIMSS and TIMSS Advanced since 1995.

## TIMSS Advanced 2015

With the current emphasis on college and career readiness and increasing global competitiveness in STEM (science, technology, engineering, and mathematics) fields, in 2015 TIMSS Advanced once again was joined with TIMSS. First conducted in 1995 as part of TIMSS and then separately again in 2008, TIMSS Advanced is the only international assessment that provides essential information about students' achievement in advanced mathematics and physics. It assesses students in their final year of secondary school (often 12<sup>th</sup> grade) who are engaged in advanced mathematics and physics studies that prepare them to enter STEM programs in higher education.

TIMSS Advanced 2015 was offered together with TIMSS 2015 to provide 20 years of achievement trends at three important points in students' schooling (4<sup>th</sup> grade, 8<sup>th</sup> grade, and final grade), and to examine how the foundations established in primary school can influence students' educational career through lower secondary and impact achievement in students' final year of secondary school. To develop the *TIMSS Advanced 2015 Assessment Frameworks*, the participating countries worked collaboratively to build upon the work of TIMSS Advanced 2008. In 2015, the advanced mathematics assessment covered algebra, calculus, and geometry (including trigonometry); the physics assessment covered mechanics and thermodynamics, electricity and

magnetism, and wave phenomena and atomic/nuclear physics. The assessments consisted of approximately 100 items each for advanced mathematics and for physics. Questionnaires were completed by the students, their teachers, and school principals.

Exhibit 1 lists the nine countries that participated in TIMSS Advanced 2015, including France, Italy, Lebanon, Norway, Portugal, the Russian Federation, Slovenia, Sweden, and the United States. In Advanced Mathematics, the Russian Federation participated with two populations of students—Profile students and a subset of those students who were in an even more intensive program. The students in the intensive program took 6 hours or more of mathematics lessons per week.

**Exhibit 1: Countries Participating in TIMSS Advanced 2015**

Country	Also Participated	
France	1995	
Italy	1995	2008
Lebanon		2008
Norway	1995	2008
Portugal		
Russian Federation*	1995	2008
Slovenia	1995	2008
Sweden	1995	2008
United States	1995	

\*For advanced mathematics, the Russian Federation participated in 2015 with an expanded population that included the more specialized students assessed in 1995 and 2008.

In total, TIMSS Advanced 2015 was administered to more than 56,000 students (32,000 in advanced mathematics and 24,000 in physics). Nearly 5,000 teachers and 3,000 schools completed questionnaires.

In shaping educational policy, countries need to consider the issue of at what level and how many specialists they should be preparing in mathematics, science, and engineering. Globally, students need to be educated to teach and pursue careers in a host of crucial medical, social, industrial, and agricultural fields. However, across countries, programs in advanced mathematics and physics vary widely in terms of the proportion of the age cohort of students enrolled in them

and in the depth and sophistication of subject matter content included. By the end of the secondary level, a significant proportion of the age cohort may no longer be in school; and for students still in school, the percentages electing to specialize in advanced mathematics and physics vary greatly. Thus, it is important to realize that TIMSS Advanced 2015 provides information on the following:

- The numbers of students and the proportion of the overall student population who are participating in advanced mathematics and physics study at the end of secondary school
- The achievement of students in programs and tracks taking advanced mathematics and physics
- A rich set of contextual data on curricula, instruction, teacher preparation, and students' future plans that can be used to guide education reform and policy planning in STEM fields

## TIMSS 2015

TIMSS is an ongoing international assessment of mathematics and science at the fourth and eighth grades that has been conducted every four years since 1995. TIMSS 2015 is the sixth in the TIMSS series, providing 20 years of trends in educational achievement in mathematics and science, together with comprehensive data on students' contexts for learning in these curricular areas.

In 2015, 57 countries and 7 benchmarking entities (regional jurisdictions of countries such as states or provinces) participated in TIMSS. In total, more than 580,000 students around the world participated in TIMSS 2015.

## Quality Assurance

TIMSS 2015 and TIMSS Advanced 2015 made every effort to attend to the quality and comparability of the data through careful planning and documentation, cooperation among participating countries, standardized procedures, and rigorous attention to quality control throughout. The assessments were given to carefully selected and well-documented probability samples of students. Staff from Statistics Canada and the IEA Data Processing and Research Center (DPC) worked with National Research Coordinators on all phases of sampling activities to ensure compliance with sampling and participation requirements, with the few exceptions from compliance annotated in the data exhibits. The IEA Secretariat worked with the TIMSS & PIRLS International Study Center to manage an extensive series of verification checks to ensure the comparability of translations of the assessment items and questionnaires, and to conduct an international quality assurance program of school visits to monitor and report on the administration of the assessment. IEA DPC staff worked closely with National Research Coordinators all through the project to organize data collection operations and to check all data for accuracy and consistency within and across countries.

## TIMSS Advanced 2015 Results

The international results for TIMSS Advanced 2015 are reported on this website, and the TIMSS 2015 results for mathematics and science achievement at fourth and eighth grades also can be accessed.

The TIMSS Advanced 2015 results are presented separately for Advanced Mathematics and Physics, with 11 chapters for each subject that contain an overview and exhibits summarizing students' achievement, on average and at the International Benchmarks, as well as exhibits describing the school and classroom contexts for students in special STEM programs or tracks in their final year of secondary school. The data exhibits can be downloaded and printed from the [Download Center](#).

The TIMSS Advanced 2015 website includes links to:

- [TIMSS Advanced 2015 Assessment Frameworks](#) describes the advanced mathematics and physics frameworks, including the major content and cognitive domains to be assessed and the information to be collected in the student, teacher, and school questionnaires
- [Methods and Procedures in TIMSS Advanced 2015](#) documents the methods and procedures used to develop, implement, and analyze the results from the TIMSS Advanced 2015 assessments

Note: All TIMSS Advanced 2015 countries participated in TIMSS 2015 and are included in the [TIMSS 2015 Encyclopedia](#). Also, considerable information about the TIMSS Advanced 2015 programs and tracks as well as the courses taken by the TIMSS Advanced students can be found in the TIMSS Advanced 2015 exhibits and the curriculum chapter.





# CHAPTER M1: STUDENT ACHIEVEMENT

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS



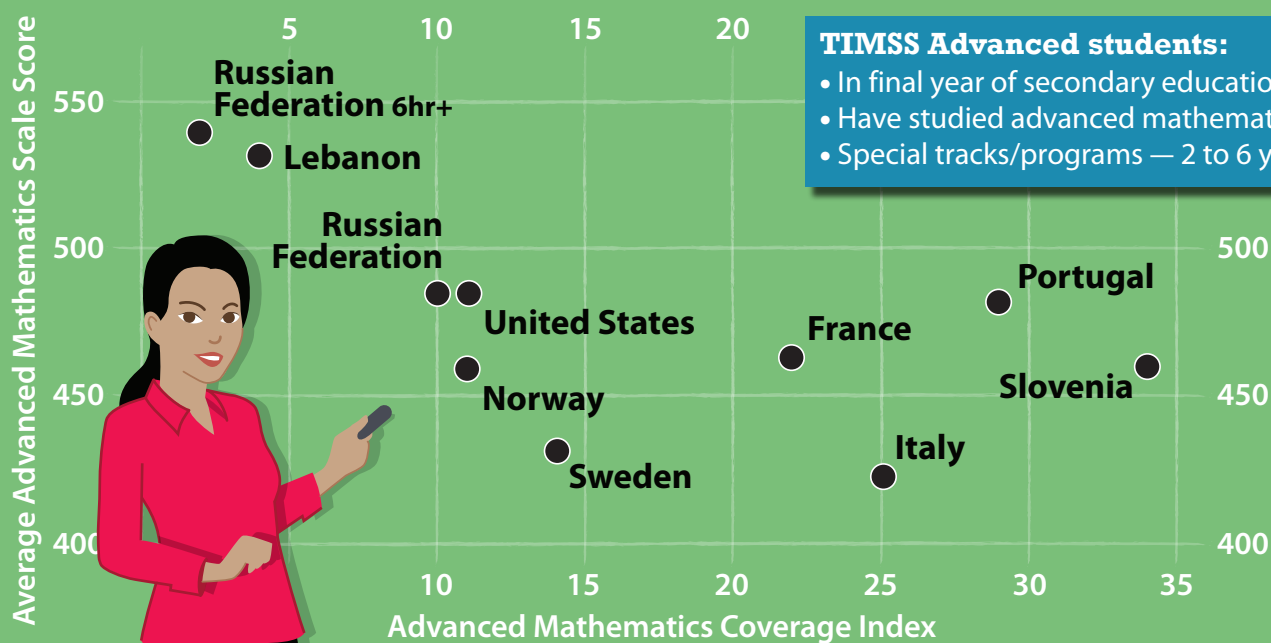
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# International Achievement in Advanced Mathematics

## Average Advanced Mathematics Achievement by Advanced Mathematics Coverage Index\*



### TIMSS Advanced students:

- In final year of secondary education
- Have studied advanced mathematics
- Special tracks/programs — 2 to 6 years



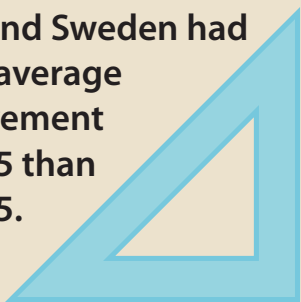
\*The TIMSS Advanced Mathematics Coverage Index quantifies the differences across countries in the percentage of students enrolled in advanced programs/tracks.

In today's technological world, countries need STEM experts. The big question: How many to educate at how high a level?

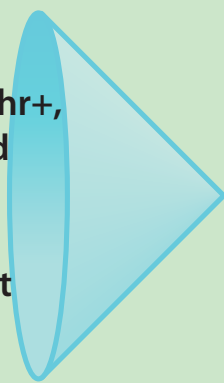
- The **2%** of Russian students in intensive study (6 hours-plus per week) and the **4%** of Lebanese students in TIMSS Advanced had the highest achievement
- The Russian Federation, with a total of **10%** of its students in TIMSS Advanced, the United States with **11%**, and Portugal with **29%** (nearly 3 times that of Russia and the U.S.) had the next highest achievement
- Norway (**11%**), France (**22%**), and Slovenia (**34%**) had comparable achievement
- Sweden (**14%**) and Italy (**25%**) had comparable achievement

## TIMSS Advanced 2015 Reveals Disappointing Trends in Mathematics Achievement

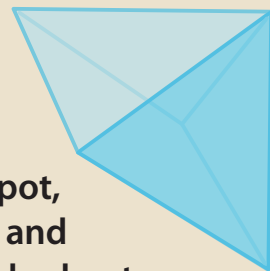
Of the 6 countries with 20-year trend data, France, Italy, and Sweden had lower average achievement in 2015 than in 1995.



The Russian Federation 6hr+, Slovenia, and the United States had no significant difference.



As a bright spot, Norway and Sweden had upturns between 2008 and 2015.



## Attracting Women to STEM Education Remains a Challenge

More **Males** than **Females** were enrolled in Advanced Mathematics programs in **6** countries.

More Males enrolled

**6** Countries  
France, Sweden, Norway, Italy, Lebanon, the United States

**2** Countries  
Slovenia, Portugal

More Females enrolled

**Males** had higher achievement than **Females** in **6** countries.

Males higher achievement

**6** Countries  
Russian Federation, Norway, Sweden, France, Slovenia, the United States

**0** Countries

Females higher achievement





**Exhibit M1.1: Structural Characteristics of the Advanced Mathematics Programs (Tracks)**

Reported by National Research Coordinators

Country	Description of How the Programs (Tracks) Fit into the Overall Curriculum	Number of Years Students are Taught in These Programs (Tracks)	Number of Hours of Advanced Mathematics Instruction per Year	Criteria for Admission to These Programs (Tracks)	Prerequisites for Admission to These Programs (Tracks)
France	Secondary schooling spans Grades 6–12. At the end of Grade 9 students choose either a vocational program or the general program. Students attending the general program choose among four tracks at the end of Grade 10—technological, literary, economic and social, or scientific. Students choosing the scientific track choose either the engineering sciences or the life and Earth sciences emphasis at Grade 11. At Grade 12, these students additionally choose a specialization among four—life and Earth sciences, mathematics, physics and chemistry, or computational sciences.	2 years	173	Students' skills and attitudes towards science, their grades in mathematics and science, and teachers' and principals' opinions and reports all contribute.	Completion of Grade 10
Italy	Secondary education can last 5 years and is given in three types of schools—lyceums, technical schools, and vocational schools. The students assessed by TIMSS Advanced 2015 were in Grade 13 and completed an advanced mathematics course or an advanced mathematics and physics course. Most of these students were in general schools with scientific focus on mathematics and physics (Liceo Scientifico), in general schools with a focus on science, mathematics and physics (Liceo Scientifico opzione Scienze Applicate), or in technical institutes and receiving full-time vocational training.	5 years	132	Completion of lower secondary education (Scuola secondaria di I grado), Grades 6–8, and success on the national examination at the end of Grade 8.	No prerequisites
Lebanon	The curriculum in Lebanon is spiral in nature so mathematical concepts are introduced in Grade 1 and accumulate until Grade 12. Participation is a prerequisite for the university specialized studies in mathematics or related studies.	6 years	250	Students must obtain a grade of 12 out of 20 or higher in mathematics in Grade 11.	Since the system is spiral, students are prepared from Grade 1 on to take the courses in advanced mathematics.
Norway	The Norwegian students assessed by TIMSS Advanced 2015 completed 10 years of compulsory education followed by 3 years of upper-secondary education. Upper-secondary education is not compulsory. However, all students have the right to an upper-secondary education. Almost the entire cohort enters this level, approximately half of them in an academic track, the other half in vocational programs. All students in the academic track must take some mathematics in Grades 11 and 12. Those who want to specialize in mathematics choose the most theoretical courses offered. The last two of these are called "Mathematics R1" and "Mathematics R2," normally taken in Grades 12 and 13, respectively. The Norwegian students assessed in advanced mathematics by TIMSS Advanced 2015 took the R2 course in their final year of secondary education.	2 years	140	Students must successfully complete a theoretical mathematics course in Grade 11.	In Grade 11 students can choose between two courses. The most theoretical one of these is a prerequisite for the R1 course. The R1 course is a prerequisite for the R2 course.
Portugal	Upper-secondary schooling is a 3-year program (Grades 10–12) and is compulsory for all students. Depending on the program in the upper-secondary academic track, students may take either 3 years of advanced mathematics (Matemática A for Sciences and Technology or Socio-Economic programs) or 2 years of Matemática B (Arts programs) with 2 years of Mathematics for the Social Sciences (Languages and Humanities programs). Only students enrolled in advanced mathematics (Matemática A) were assessed in TIMSS Advanced 2015.	3 years	146	Completion of lower secondary education. In upper-secondary education, students can choose a secondary education study program according to their academic and/or professional interests.	No prerequisites

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M1.1: Structural Characteristics of the Advanced Mathematics Programs (Tracks) (Continued)**

Country	Description of How the Programs (Tracks) Fit into the Overall Curriculum	Number of Years Students are Taught in These Programs (Tracks)	Number of Hours of Advanced Mathematics Instruction per Year	Criteria for Admission to These Programs (Tracks)	Prerequisites for Admission to These Programs (Tracks)
Russian Federation	<p>Since 2012, for their final two years of secondary school students in Grade 10 are divided into three streams that include different amounts of mathematics courses: Basic—3 hours per week; Profile—4.5 hours per week; and Intensive—6 or more hours per week. The courses also vary in depth of content and attainment requirements. Grade 11 students in both the Profile and Intensive streams participated in TIMSS Advanced 2015. The students in the Profile and Intensive streams study in lyceums, gymnasiums, special schools for mathematics and physics tracks, and general secondary schools with different streams at the upper-secondary level. In Grade 11 they have mastered the general mathematics courses in Grades 1–9 before moving on to more advanced courses in Grades 10 and 11. In some cases, advanced coursework in mathematics is available for these students in Grades 7–9, so Profile and Intensive stream classes may consist of students who have done advanced work in mathematics in previous courses. Students who successfully complete Profile and Intensive programs of mathematics study meet the requirements for admission to universities that require a sufficiently deep knowledge of mathematics.</p>	2 years	<p>158 (Profile)</p> <p>210 (Intensive)</p>	Successful completion of basic education (Grades 1–9), successfully passing the state mathematics examination at Grade 9, and possibly an interview, oral or written mathematics test or examination, organized by the student's school.	No prerequisites
Slovenia	<p>Secondary education consists of two types of programs: general gymnasias; and vocational or technically oriented programs. Only the general gymnasias program offers students the possibility of admission to university studies. All general gymnasias students study the same mathematics course during their 4-year program.</p>	4 years	105	Completion of elementary schooling.	No prerequisites

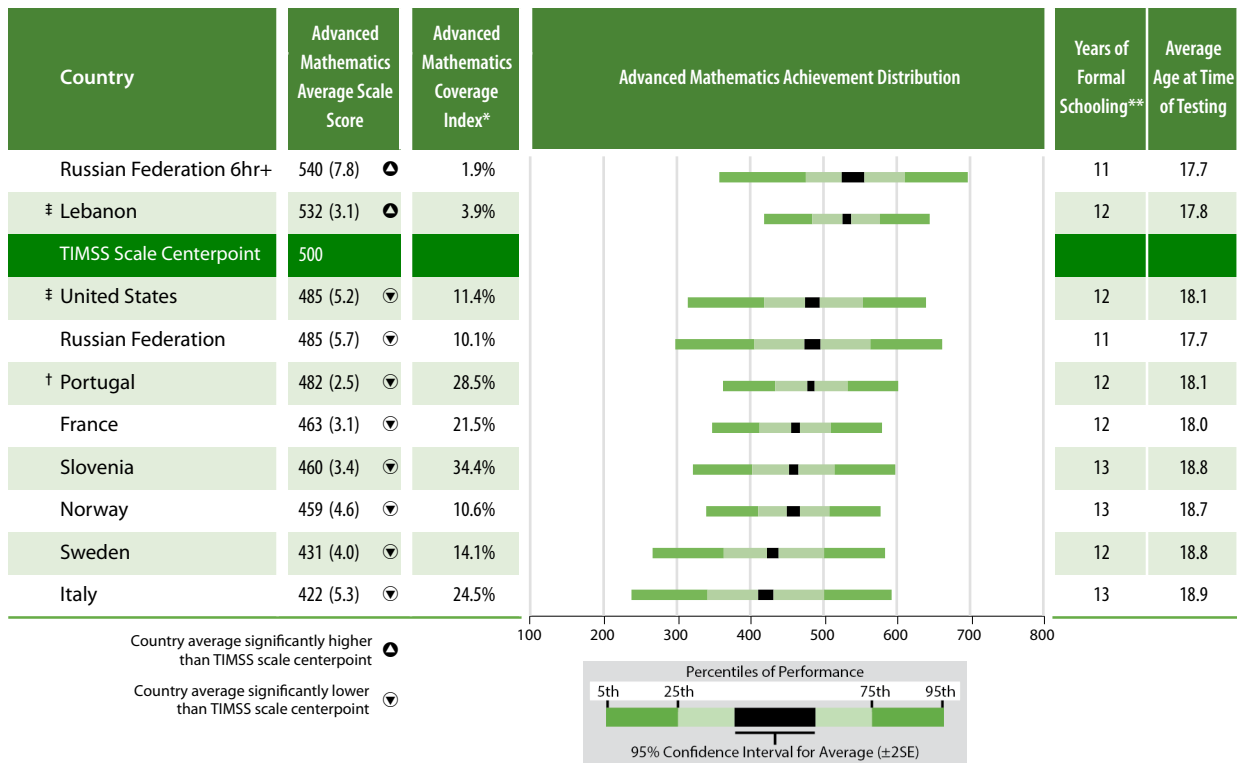
SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M1.1: Structural Characteristics of the Advanced Mathematics Programs (Tracks) (Continued)**

Country	Description of How the Programs (Tracks) Fit into the Overall Curriculum	Number of Years Students are Taught in These Programs (Tracks)	Number of Hours of Advanced Mathematics Instruction per Year	Criteria for Admission to These Programs (Tracks)	Prerequisites for Admission to These Programs (Tracks)
Sweden	Upper-secondary education starts at Grade 10 and is divided into 18 national 3-year programs. There are 12 vocational programs and 6 programs preparing for studies at the university level. In Swedish upper-secondary schools, mathematics is taught in consecutive courses at 5 levels—Mathematics 1, 2, 3, 4, and 5—and in one specialized course. In addition, courses at the first 2 levels are taught in 3 tracks with one track for vocational programs, one for social science and economics programs, and one for science and technology programs. The third level has 2 tracks (no track for vocational programs) and there is only one track in levels 4 and 5. The vast majority of students studying mathematics at level 4 or above are found in the science and technology programs. For the science program, most students study Mathematics 4. It is compulsory for the vast majority of students. For students in the technology program, Mathematics 4 is compulsory in one track of the program and optional for students within the other tracks, but it is chosen by many students. Students who participated in TIMSS Advanced 2015 in advanced mathematics completed either Mathematics 1–4 (400 credits) or Mathematics 1–3, and were about to complete Mathematics 4. Some of the students who completed Mathematics 4 completed or were taking Mathematics 5 (100 credits) and/or a mathematics specialized course. These students studied in either the natural science program or technology program at Grade 12.	3 years	Varying, but approximately 150 on average	Completion of 9-year compulsory school with passing grades in Swedish, English, mathematics, biology, physics, chemistry, and at least six other subjects.	No prerequisites
United States	The mathematics programs/tracks vary by state and district. All students begin studying mathematics in elementary school with a focus on basic arithmetic and learning about objects they encounter in the environment. In middle school, students study basic algebra and concepts of variables, integers and polynomials. Some students take more advanced algebra in middle school. In high school, most students start taking focused courses such as higher level algebra, geometry, and pre-calculus. After completing those secondary mathematics requirements students can begin studying advanced mathematics (calculus/statistics) courses. The year during which students begin studying advanced courses varies, but generally it is in grade 11 and 12. In advanced mathematics, there are two main programs that are used across many states: College Board's Advanced Placement (AP) Program and the International Baccalaureate's (IB) Diploma Programme. The AP Calculus program includes two calculus courses, AP Calculus AB and AP Calculus BC, for students to choose between. Each course is independent and designed to be taught for one full academic year. AP Calculus BC is an accelerated version of the AB course that also covers additional topics. IB Mathematics is a two-year comprehensive program that also offers two courses, Standard Level (SL) and High Level (HL), for students to choose between. Each course is independent and has a two year duration. The TIMSS Advanced mathematics sample includes Grade 12 students who have taken an advanced mathematics course (AP, IB, or another advanced mathematics course specific to their state/district) in Grade 12 or in a prior grade.	Varies by school and by course	Varies by school and by course	Varies by district and school	Varies by school and by course

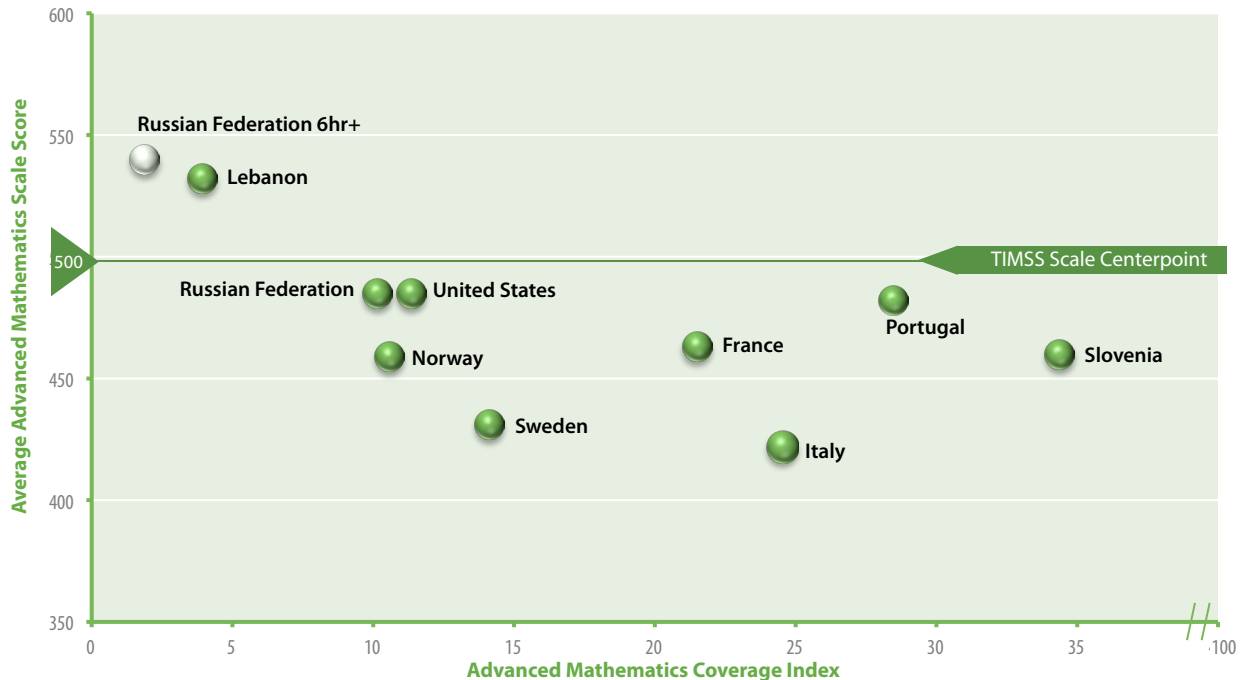
SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M1.2: Distribution of Advanced Mathematics Achievement**



\* See Appendix MC.2 for a description of the Advanced Mathematics Coverage Index.  
 \*\* Represents years of schooling counting from first year of primary or basic education (first year of ISCED Level 1).  
 The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.  
 The TIMSS Advanced achievement scale was established in 1995 based on the combined achievement distribution of all countries that participated in TIMSS Advanced 1995. To provide a point of reference for country comparisons, the scale centerpoint of 500 was located at the mean of the combined achievement distribution. The units of the scale were chosen so that 100 scale score points corresponded to the standard deviation of the distribution.  
 See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.  
 ( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Average Advanced Mathematics Achievement by Advanced Mathematics Coverage Index\***





**Exhibit M1.3: Multiple Comparisons of Average Advanced Mathematics Achievement**

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

Country	Average Scale Score	Russian Federation 6hr+	Lebanon	United States	Russian Federation	Portugal	France	Slovenia	Norway	Sweden	Italy
		Russian Federation 6hr+	540 (7.8)			▲	▲	▲	▲	▲	▲
Lebanon	532 (3.1)			▲	▲	▲	▲	▲	▲	▲	
United States	485 (5.2)	▼	▼				▲	▲	▲	▲	
Russian Federation	485 (5.7)	▼	▼				▲	▲	▲	▲	
Portugal	482 (2.5)	▼	▼				▲	▲	▲	▲	
France	463 (3.1)	▼	▼	▼	▼	▼				▲	
Slovenia	460 (3.4)	▼	▼	▼	▼	▼				▲	
Norway	459 (4.6)	▼	▼	▼	▼	▼				▲	
Sweden	431 (4.0)	▼	▼	▼	▼	▼	▼	▼	▼		
Italy	422 (5.3)	▼	▼	▼	▼	▼	▼	▼	▼		

- ▲ Average achievement significantly higher than comparison country
- ▼ Average achievement significantly lower than comparison country

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

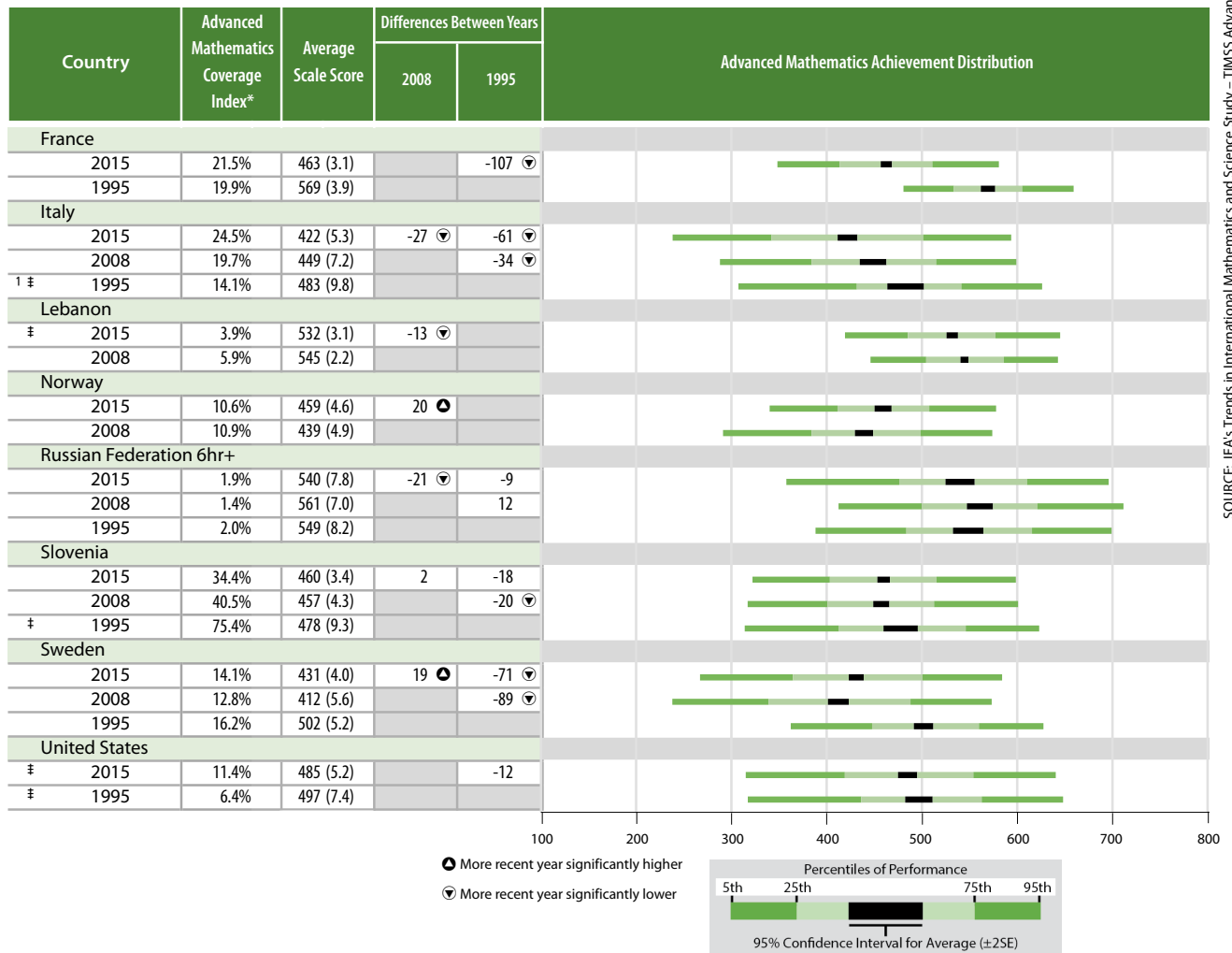
Significance tests were not adjusted for multiple comparisons. Five percent of the comparisons would be statistically significant by chance alone.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M1.4: Differences in Advanced Mathematics Achievement Across Assessment Years**

Instructions: Read across the row to determine if the performance in the row year is significantly higher (▲) or significantly lower (▼) than the performance in the column year.



SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

\* See Appendix MC.2 for a description of the Advanced Mathematics Coverage Index.

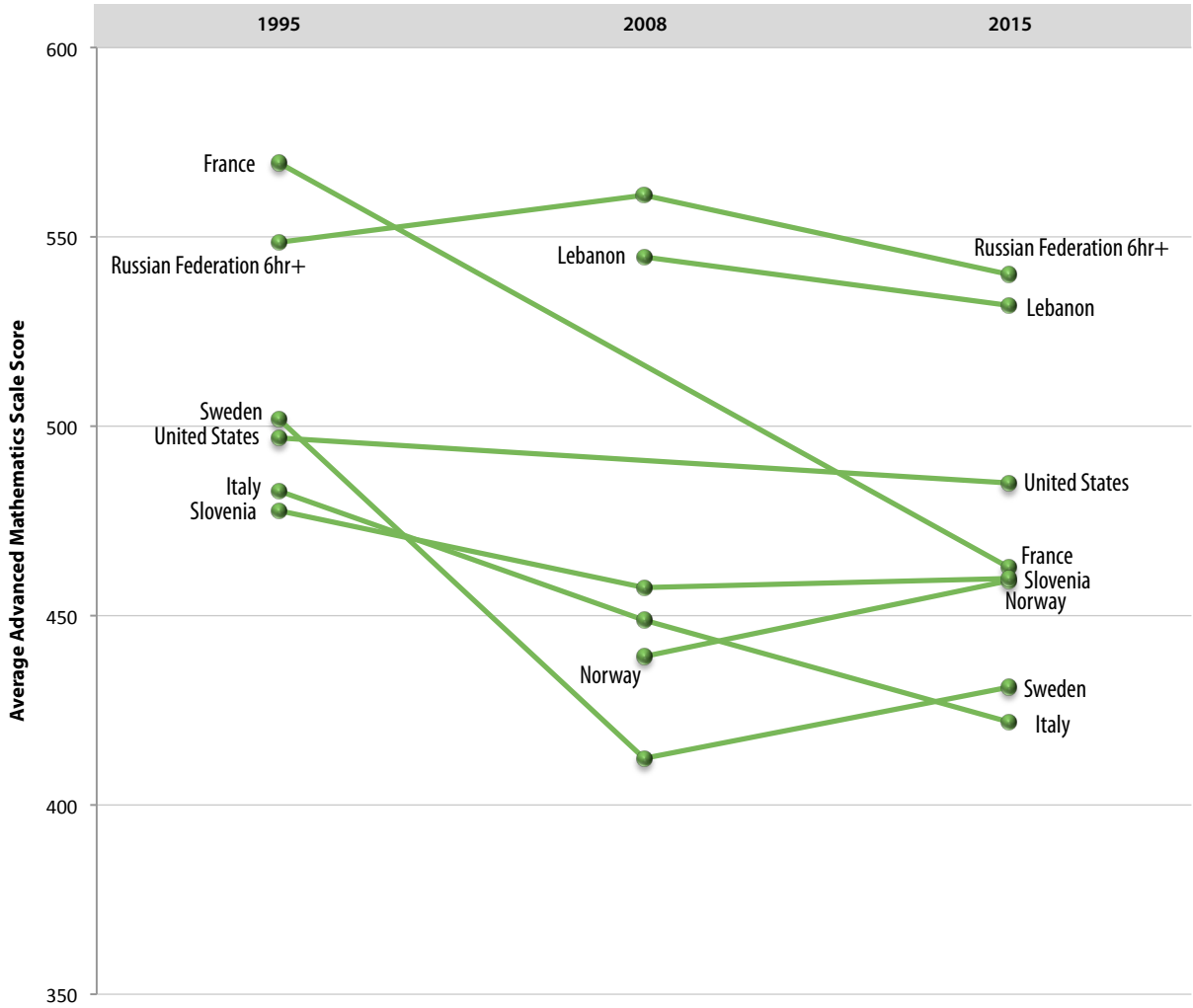
Russian Federation trend results are available only for the Intensive stream students (6hr+). The United States adjusted the 1995 sample to correspond with the course-taking definitions used in 2015, and the 1995 results were recomputed.

See Appendix MC.1 for target population coverage notes 1, 2, and 3.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and §.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M1.4: Differences in Advanced Mathematics Achievement Across Assessment Years (Continued)**

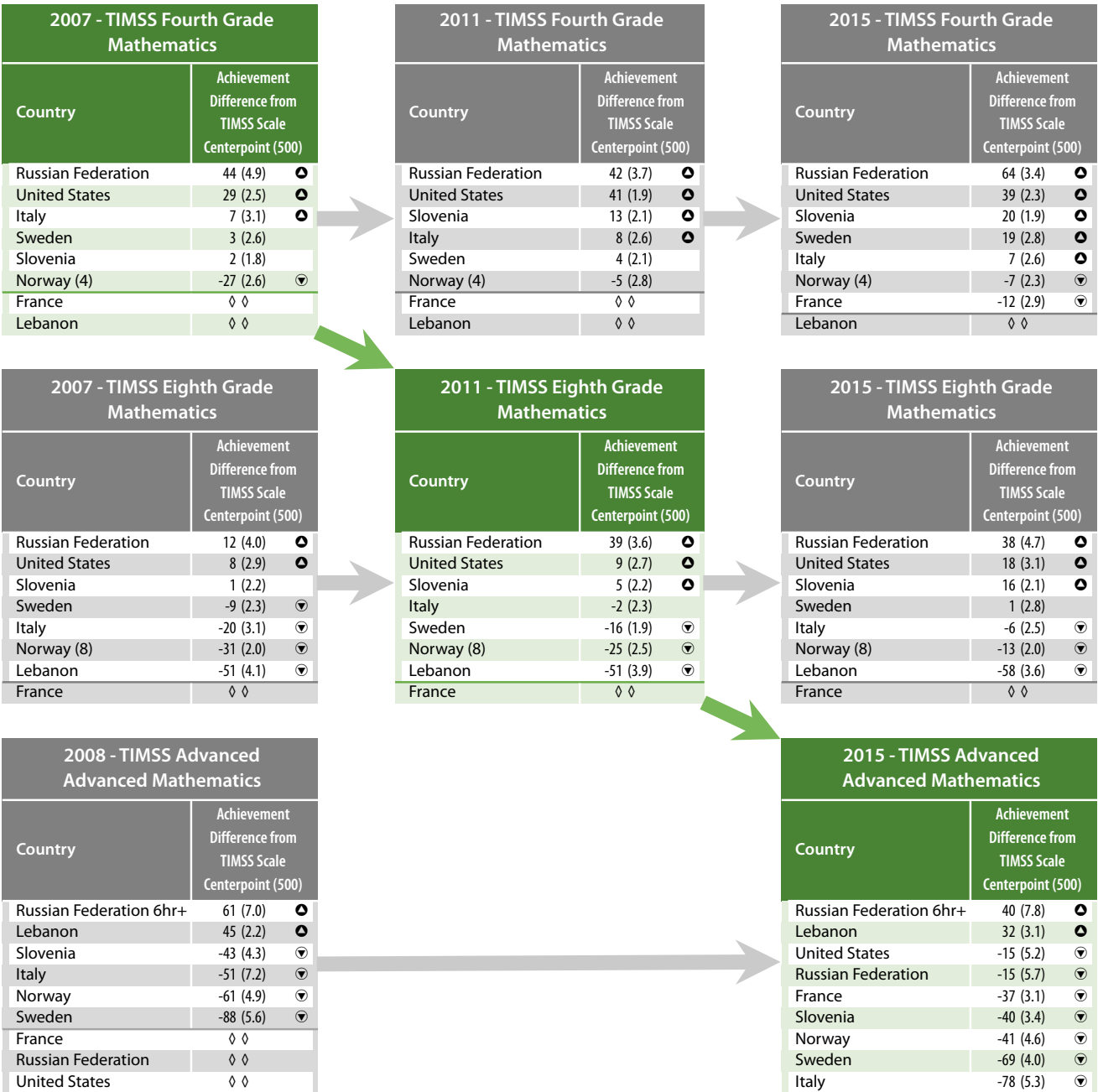


SOURCE: IEA's Trends in International Mathematics and Science Study - TIMSS Advanced 2015

Russian Federation trend results are available only for the Intensive stream students (6hr+). The United States adjusted the 1995 sample to correspond with the course-taking definitions used in 2015, and the 1995 results were recomputed.

**Exhibit M1.5: Relative Achievement of 2015 Advanced Mathematics Cohort at the Eighth and Fourth Grades\***

Instructions: To compare relative achievement across grades as the cohort of students assessed at the fourth grade in 2007 moved to eighth grade four years later in 2011 and then to TIMSS Advanced in 2015, start in the upper-left hand panel and follow the darker green arrows pointing diagonally downwards.



SOURCE: IEA's Trends in International Mathematics and Science Study - TIMSS Advanced 2015

- ▲ Country average significantly higher than the centerpoint of the TIMSS scale
- ▼ Country average significantly lower than the centerpoint of the TIMSS scale

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

Russian Federation trend results are available only for the Intensive stream students (6hr+). The United States adjusted the 1995 sample to correspond with the course-taking definitions used in 2015, and the 1995 results were recomputed.

A diamond (◊) indicates the country did not participate in this year's assessment.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

\* TIMSS 2007 data from: *TIMSS 2007 International Mathematics Report*

TIMSS 2011 data from: *TIMSS 2011 International Results in Mathematics*

TIMSS 2015 data from: *TIMSS 2015 International Results in Mathematics*

TIMSS Advanced 2008 data from: *TIMSS Advanced 2008 International Report*

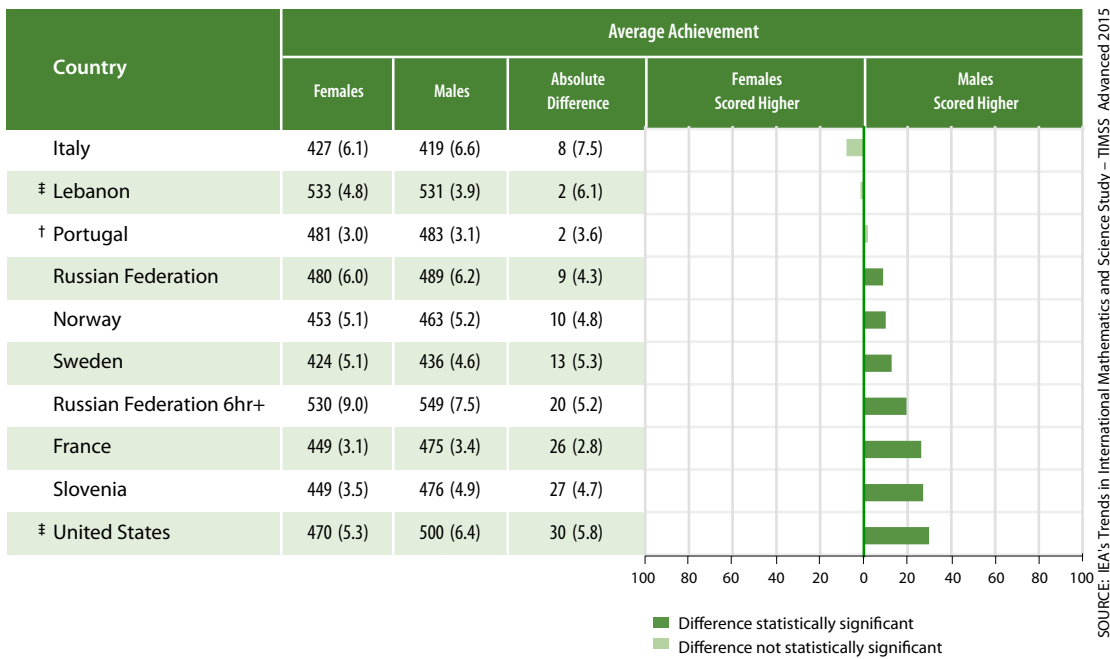


**Exhibit M1.6: Advanced Mathematics Participation and Average Achievement by Gender**

**Participation in Advanced Mathematics by Gender**



**Average Advanced Mathematics Achievement by Gender**



The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M1.7: Differences in Advanced Mathematics Achievement by Gender Across Assessment Years**

Instructions: Read across the row to determine if the performance in the row year is significantly higher (▲) or significantly lower (▼) than the performance in the column year.

Country	Females				Males			
	Percent of Students	Average Scale Score	Differences Between Years		Percent of Students	Average Scale Score	Differences Between Years	
			2008	1995			2008	1995
<b>France</b>								
2015	47 (1.1)	449 (3.1)		-112 ▼	53 (1.1)	475 (3.4)		-100 ▼
1995	37 (2.0)	561 (4.8)			63 (2.0)	575 (4.6)		
<b>Italy</b>								
2015	37 (1.3)	427 (6.1)	-27 ▼	-50 ▼	63 (1.3)	419 (6.6)	-27 ▼	-68 ▼
2008	34 (2.5)	454 (9.5)		-23	66 (2.5)	446 (8.3)		-41 ▼
<sup>1</sup> ‡ 1995	39 (3.8)	477 (12.5)			61 (3.8)	487 (11.6)		
<b>Lebanon</b>								
<sup>‡</sup> 2015	36 (2.0)	533 (4.8)	-21 ▼		64 (2.0)	531 (3.9)	-10 ▼	
2008	29 (1.6)	554 (3.1)			71 (1.6)	541 (2.5)		
<b>Norway</b>								
2015	38 (1.4)	453 (5.1)	19 ▲		62 (1.4)	463 (5.2)	21 ▲	
2008	38 (1.7)	434 (5.3)			62 (1.7)	442 (5.6)		
<b>Russian Federation 6hr+</b>								
2015	46 (1.1)	530 (9.0)	-21	4	54 (1.1)	549 (7.5)	-20	-21
2008	45 (1.8)	551 (7.5)		25 ▲	55 (1.8)	569 (7.3)		0
1995	48 (2.4)	526 (9.1)			52 (2.4)	570 (8.7)		
<b>Slovenia</b>								
2015	60 (1.1)	449 (3.5)	1	-20	40 (1.1)	476 (4.9)	4	-10
2008	60 (1.8)	448 (5.3)		-21	40 (1.8)	472 (4.7)		-14
<sup>‡</sup> 1995	50 (4.2)	469 (11.4)			50 (4.2)	486 (11.1)		
<b>Sweden</b>								
2015	40 (1.2)	424 (5.1)	20 ▲	-68 ▼	60 (1.2)	436 (4.6)	18 ▲	-70 ▼
2008	40 (2.1)	404 (6.6)		-88 ▼	60 (2.1)	418 (6.5)		-88 ▼
1995	31 (3.5)	492 (4.8)			69 (3.5)	506 (6.9)		
<b>United States</b>								
<sup>‡</sup> 2015	49 (0.9)	470 (5.3)		-16	51 (0.9)	500 (6.4)		-7
<sup>‡</sup> 1995	47 (3.2)	486 (10.1)			53 (3.2)	507 (7.6)		

▲ More recent year significantly higher

▼ More recent year significantly lower

Russian Federation trend results are available only for the Intensive stream students (6hr+). The United States adjusted the 1995 sample to correspond with the course-taking definitions used in 2015, and the 1995 results were recomputed.

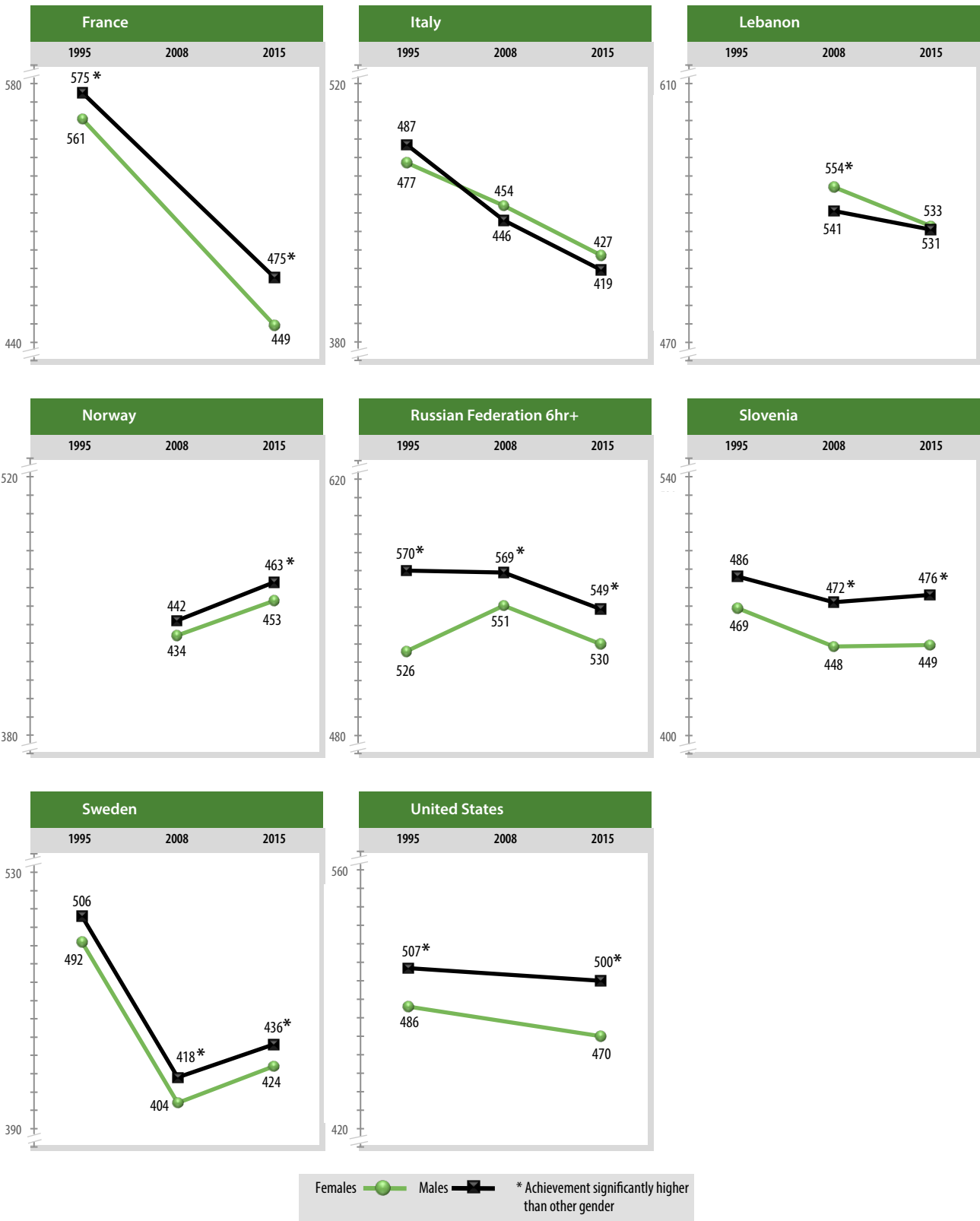
See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and §.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M1.7: Differences in Advanced Mathematics Achievement by Gender Across Assessment Years (Continued)**

**Trends in Advanced Mathematics Achievement by Gender**



SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

Russian Federation trend results are available only for the Intensive stream students (6hr+). The United States adjusted the 1995 sample to correspond with the course-taking definitions used in 2015, and the 1995 results were recomputed.

Scale interval is 10 points for each country, but the part of the scale shown differs according to each country's average achievement.



# CHAPTER M2: PERFORMANCE AT INTERNATIONAL BENCHMARKS

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS



**IEA**

**TIMSS & PIRLS**  
International Study Center  
Lynch School of Education, Boston College





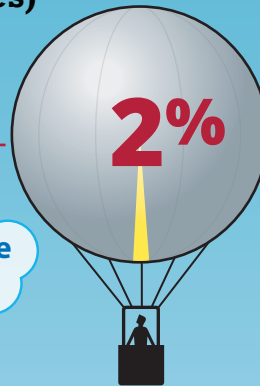
# Students Struggle to Reach the TIMSS Advanced International Benchmarks

TIMSS Advanced describes achievement at three International Benchmarks along the scale: Advanced, High, and Intermediate. There was a range of results across countries, but on average the majority of students found the TIMSS Advanced mathematics assessment very difficult.

## Percentage of Students Reaching Benchmarks (averaged across countries)

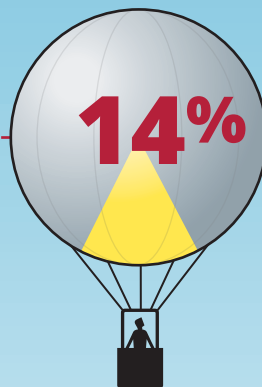
**Advanced Benchmark (625)**

In the Russian Federation, Lebanon, and the United States 7-10% reached the Advanced Benchmark, but only 1-3% did in the other countries.



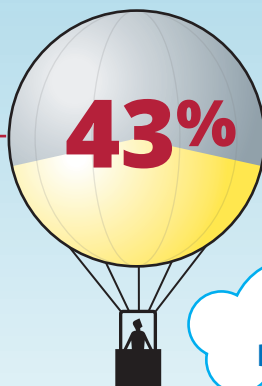
Students demonstrate thorough understanding of concepts, mastery of procedures, and mathematical reasoning skills. They can solve problems in complex contexts in algebra, calculus, geometry, and trigonometry.

**High Benchmark (550)**



Students can apply a broad range of mathematical concepts and procedures in algebra, calculus, geometry, and trigonometry to analyze and solve multi-step problems set in routine and non-routine contexts.

**Intermediate Benchmark (475)**



The surprisingly low percentages of students reaching the Intermediate Benchmark reflected substantial declines in 4 countries compared to 1995.

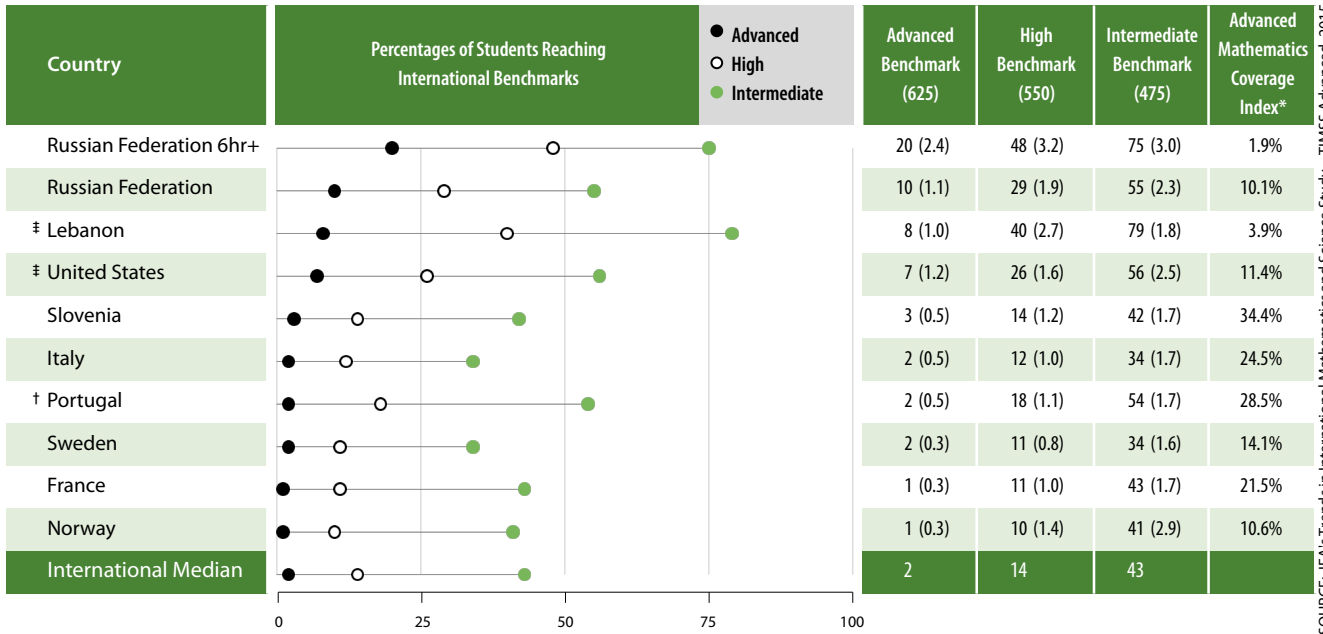
Students demonstrate basic knowledge of concepts and procedures in algebra, calculus, and geometry to solve routine problems.



**Exhibit M2.1: Descriptions of the TIMSS Advanced 2015 International Benchmarks of Advanced Mathematics Achievement**

625	Advanced International Benchmark	●
	<p><i>Students demonstrate thorough understanding of concepts, mastery of procedures, and mathematical reasoning skills. They can solve problems in complex contexts in algebra, calculus, geometry, and trigonometry.</i></p> <p>In algebra, students can reason with functions to solve pure mathematical problems. They demonstrate facility with complex numbers and permutations and can find sums of algebraic and infinite geometric series.</p> <p>In calculus, students demonstrate thorough understanding of continuity and differentiability. They can solve problems about optimization in different contexts and justify their solutions. They can use definite integrals to calculate the area between two curves.</p> <p>Students use geometric reasoning to solve complex problems. They use properties of vectors to express relationships among vectors. They can use trigonometric properties including the sine and cosine rules to solve non-routine problems about geometric figures.</p>	
550	High International Benchmark	○
	<p><i>Students can apply a broad range of mathematical concepts and procedures in algebra, calculus, geometry, and trigonometry to analyze and solve multi-step problems set in routine and non-routine contexts.</i></p> <p>Students can analyze and solve algebra problems, including problems set in a practical context. They can solve problems requiring interpretation of information related to functions and graphs of functions. They can determine a sum of an arithmetic sequence and solve quadratic and other inequalities. They can simplify logarithmic expressions and multiply complex numbers.</p> <p>In calculus, students have a basic understanding of continuity and differentiability. They can analyze equations of functions and graphs of functions. They can relate the graphs of functions to graphs and signs of their first and second derivatives. Students show some conceptual understanding of definite integrals.</p> <p>Students can use trigonometric properties to solve a variety of problems involving trigonometric functions and geometric figures. They can use the Cartesian plane to solve problems, identify a vector perpendicular to a given vector, and prove that a quadrilateral given in the coordinate system is a parallelogram.</p>	
475	Intermediate International Benchmark	●
	<p><i>Students demonstrate basic knowledge of concepts and procedures in algebra, calculus, and geometry to solve routine problems.</i></p> <p>Students can apply and transform a formula to solve a word problem. They can determine a term in a geometric sequence and analyze a proposed solution of a simple logarithmic equation. They can recognize a graph of the absolute value of a function and identify and evaluate composite functions.</p> <p>Students can find the derivative of exponential, trigonometric, and simple rational functions. They can find limits of rational and exponential functions. They can make connections between the sign of the derivative and the graph of a function.</p> <p>Students can use knowledge of basic properties of geometric figures and the Pythagorean theorem to solve problems. They can add and subtract vectors in coordinate form.</p>	

**Exhibit M2.2: Performance at the International Benchmarks of Advanced Mathematics Achievement**



SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

\* See Appendix MC.2 for a description of the Advanced Mathematics Coverage Index.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.3: Percentages of Students Reaching the International Benchmarks of Advanced Mathematics Achievement Across Assessment Years**

Country	Advanced International Benchmark (625)			High International Benchmark (550)			Intermediate International Benchmark (475)		
	Percent of Students			Percent of Students			Percent of Students		
	2015	2008	1995	2015	2008	1995	2015	2008	1995
Russian Federation 6hr+	20	24	22	48	55	51	75	83	78
Lebanon	8	9		40	47		79	88	
United States	7		8	26		30	56		62
Slovenia	3	3	5	14	14	23	42	41	54
Italy	2	3	5	12	14	22	34	41	59
Sweden	2	1	6	11	9	30	34	29	64
France	1		15	11		64	43		96
Norway	1	1		10	9		41	35	

- 2015 percent significantly higher
- ▼ 2015 percent significantly lower

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

Russian Federation trend results are available only for the Intensive stream students (6hr+). The United States adjusted the 1995 sample to correspond with the course-taking definitions used in 2015, and the 1995 results were recomputed.  
An empty cell indicates a country did not participate in that year's assessment.



**Exhibit M2.4: Description of the TIMSS Advanced 2015 Intermediate International Benchmark (475) of Advanced Mathematics Achievement**

**475 Intermediate International Benchmark**

**Summary**

*Students demonstrate basic knowledge of concepts and procedures in algebra, calculus, and geometry to solve routine problems.*

Students can apply and transform a formula to solve a word problem. They can determine a term in a geometric sequence and analyze a proposed solution of a simple logarithmic equation. They can recognize a graph of the absolute value of a function and identify and evaluate composite functions.

Students can find the derivative of exponential, trigonometric, and simple rational functions. They can find limits of rational and exponential functions. They can make connections between the sign of the derivative and the graph of a function.

Students can use knowledge of basic properties of geometric figures and the Pythagorean theorem to solve problems. They can add and subtract vectors in coordinate form.

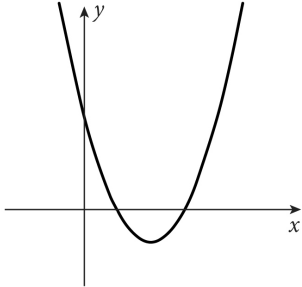
SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M2.4.1: Intermediate International Benchmark – Example Item 1**

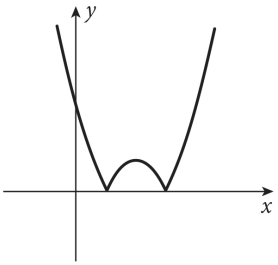
Country	Percent Correct
Slovenia	88 (1.6) ●
† Portugal	86 (1.4) ●
Russian Federation 6hr+	84 (2.2) ●
Russian Federation	71 (2.2) ●
‡ Lebanon	70 (2.9)
International Avg.	65 (0.7)
France	62 (1.5) ▼
Italy	60 (2.5) ▼
Norway	54 (2.1) ▼
‡ United States	54 (2.5) ▼
Sweden	43 (2.8) ▼

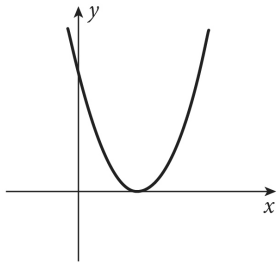
Content Domain: Algebra  
 Cognitive Domain: Knowing  
 Description: Recognizes the graph of the absolute value of a function given the graph of the function

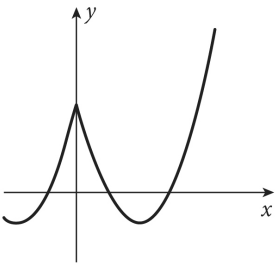
The graph of  $y = f(x)$  is shown here.

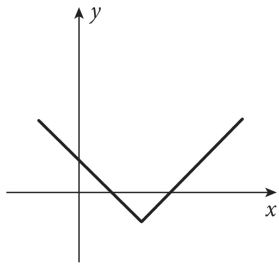


Which one of the following graphs shows  $y = |f(x)|$  ?

● 

ⓑ 

ⓒ 

ⓓ 

- Percent significantly higher than international average
- ▼ Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M2.4.1: Intermediate International Benchmark – Example Item 1  
(Continued)**

Country	Percent of Students Responding to Each Answer Option				
	A	B	C	D	NR*
Slovenia	88 (1.6)	6 (1.1)	4 (0.7)	1 (0.3)	0 (0.3)
† Portugal	86 (1.4)	7 (1.0)	2 (0.4)	5 (0.7)	0 (0.2)
Russian Federation 6hr+	84 (2.2)	3 (0.8)	6 (1.9)	6 (1.4)	0 (0.1)
Russian Federation	71 (2.2)	12 (1.5)	6 (0.9)	10 (1.2)	0 (0.1)
‡ Lebanon	70 (2.9)	21 (2.6)	5 (1.3)	3 (1.1)	1 (0.4)
France	62 (1.5)	23 (1.3)	2 (0.4)	12 (0.9)	1 (0.3)
Italy	60 (2.5)	25 (2.2)	8 (1.1)	4 (0.7)	3 (0.7)
Norway	54 (2.1)	29 (1.6)	6 (0.7)	9 (1.1)	3 (0.4)
‡ United States	54 (2.5)	14 (1.7)	1 (0.5)	29 (2.0)	1 (0.5)
Sweden	43 (2.8)	20 (1.6)	9 (1.0)	26 (1.9)	1 (0.3)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS-Advanced 2015

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.4.2: Intermediate International Benchmark – Example Item 2**

Country	Percent Correct	Content Domain: Algebra
		Cognitive Domain: Knowing
		Description: Determines which term has a given value in a geometric sequence
Russian Federation 6hr+	79 (1.9) ▲	<p>In the geometric sequence <math>\frac{1}{3}, 1, 3, \dots, t_n, \dots</math>, where <math>t_n</math> is the <math>n^{\text{th}}</math> term, which term has a value of 243?</p> <p>Ⓐ <math>t_6</math></p> <p>● <math>t_7</math></p> <p>Ⓒ <math>t_8</math></p> <p>Ⓓ <math>t_{81}</math></p>
Russian Federation	71 (1.8) ▲	
‡ United States	61 (2.0) ▲	
Norway	59 (2.2) ▲	
Slovenia	57 (2.4) ▲	
International Avg.	50 (0.7)	
‡ Lebanon	47 (3.7)	
† Portugal	45 (1.7) ▼	
Sweden	45 (1.3) ▼	
Italy	36 (1.8) ▼	
France	34 (1.4) ▼	

- ▲ Percent significantly higher than international average
- ▼ Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M2.4.2: Intermediate International Benchmark – Example Item 2 (Continued)**

Country	Percent of Students Responding to Each Answer Option				
	A	B	C	D	NR*
Russian Federation 6hr+	9 (1.1)	79 (1.9)	3 (0.6)	8 (1.2)	1 (0.2)
Russian Federation	11 (1.0)	71 (1.8)	5 (0.5)	13 (1.4)	1 (0.2)
‡ United States	14 (1.4)	61 (2.0)	8 (1.1)	12 (1.2)	6 (2.1)
Norway	15 (1.3)	59 (2.2)	7 (1.1)	16 (1.4)	3 (0.6)
Slovenia	11 (1.0)	57 (2.4)	8 (1.2)	19 (1.7)	5 (0.9)
‡ Lebanon	24 (2.7)	47 (3.7)	4 (0.9)	19 (2.5)	6 (1.0)
† Portugal	10 (1.2)	45 (1.7)	12 (1.0)	24 (1.6)	9 (0.8)
Sweden	14 (1.0)	45 (1.3)	12 (1.0)	24 (1.2)	6 (0.8)
Italy	13 (1.4)	36 (1.8)	11 (1.2)	27 (1.7)	14 (1.3)
France	31 (1.5)	34 (1.4)	5 (0.7)	26 (1.5)	3 (0.5)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015












\* No Response.



The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.4.3: Intermediate International Benchmark – Example Item 3**

Country	Percent Correct	Content Domain: Geometry
		Cognitive Domain: Knowing
		Description: Calculates the difference between vectors in coordinate form
France	80 (1.1) 	<p>Find the difference <math>\vec{b} - \vec{a}</math> of the vectors <math>\vec{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}</math> and <math>\vec{b} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}</math>.</p> <p>(A) <math>\begin{pmatrix} -4 \\ -2 \end{pmatrix}</math></p> <p> <math>\begin{pmatrix} -4 \\ 1 \end{pmatrix}</math></p> <p>(C) <math>\begin{pmatrix} 4 \\ -1 \end{pmatrix}</math></p> <p>(D) <math>\begin{pmatrix} 4 \\ 2 \end{pmatrix}</math></p> <p>(E) <math>\begin{pmatrix} 4 \\ 5 \end{pmatrix}</math></p>
Norway	79 (1.7) 	
Russian Federation 6hr+	77 (1.9) 	
Russian Federation	74 (1.7) 	
‡ Lebanon	72 (2.6) 	
† Portugal	71 (1.5) 	
International Avg.	62 (0.6)	
‡ United States	59 (2.6) 	
Slovenia	47 (1.7) 	
Sweden	37 (1.4) 	
Italy	37 (1.7) 	

-  Percent significantly higher than international average
-  Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and £.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015



**Exhibit M2.4.3: Intermediate International Benchmark – Example Item 3 (Continued)**

Country	Percent of Students Responding to Each Answer Option					
	A	B	C	D	E	NR*
France	1 (0.2)	80 (1.1)	12 (0.9)	1 (0.2)	1 (0.3)	5 (0.7)
Norway	1 (0.2)	79 (1.7)	14 (1.4)	1 (0.3)	1 (0.4)	4 (0.7)
Russian Federation 6hr+	1 (0.2)	77 (1.9)	18 (1.4)	1 (0.3)	2 (0.3)	2 (0.4)
Russian Federation	1 (0.3)	74 (1.7)	20 (1.4)	1 (0.4)	3 (0.6)	2 (0.3)
‡ Lebanon	2 (1.1)	72 (2.6)	13 (2.1)	2 (0.3)	2 (0.7)	9 (1.9)
† Portugal	1 (0.2)	71 (1.5)	22 (1.1)	1 (0.3)	1 (0.2)	5 (0.7)
‡ United States	2 (0.7)	59 (2.6)	30 (2.2)	2 (0.6)	3 (0.9)	4 (1.0)
Slovenia	4 (0.7)	47 (1.7)	32 (1.9)	8 (1.1)	3 (0.5)	6 (0.9)
Sweden	4 (0.6)	37 (1.4)	29 (1.0)	9 (0.8)	7 (0.9)	14 (1.2)
Italy	3 (0.7)	37 (1.7)	28 (1.7)	7 (1.0)	5 (0.7)	20 (1.3)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS-Advanced 2015

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.4.4: Intermediate International Benchmark – Example Item 4**

Country	Percent Correct
Sweden	65 (1.3) ●
† Portugal	65 (1.7) ●
Norway	63 (1.7) ●
Russian Federation 6hr+	62 (2.9)
‡ Lebanon	61 (2.3)
Slovenia	59 (2.0)
International Avg.	58 (0.6)
‡ United States	58 (2.5)
Russian Federation	53 (2.1) ▼
France	52 (1.5) ▼
Italy	50 (1.8) ▼

Content Domain: Geometry  
Cognitive Domain: Applying  
Description: Solves a word problem about height given the distance and angle of elevation

A lighthouse is located on the top of an islet. The base is 4 meters above sea level. A ship is located 170 m from the lighthouse. The angle between sea level and a straight line from the boat to the top of the lighthouse is equal to  $10^\circ$ . What is the height of the lighthouse to the nearest meter?

(A) 22 m  
 (B) 26 m  
 (C) 30 m  
 (D) 34 m

- Percent significantly higher than international average
- ▼ Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and £.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.4.4: Intermediate International Benchmark – Example Item 4 (Continued)**

Country	Percent of Students Responding to Each Answer Option				
	A	B	C	D	NR*
Sweden	5 (0.7)	65 (1.3)	18 (1.0)	5 (0.7)	7 (0.8)
† Portugal	5 (0.6)	65 (1.7)	16 (1.2)	8 (1.0)	7 (0.7)
Norway	4 (0.8)	63 (1.7)	19 (1.3)	5 (0.7)	8 (1.1)
Russian Federation 6hr+	9 (1.3)	62 (2.9)	14 (1.4)	7 (0.9)	8 (1.2)
‡ Lebanon	2 (0.9)	61 (2.3)	18 (1.8)	6 (1.4)	11 (1.9)
Slovenia	4 (0.6)	59 (2.0)	21 (1.6)	5 (0.9)	11 (1.1)
‡ United States	5 (0.9)	58 (2.5)	20 (1.8)	8 (0.9)	10 (2.3)
Russian Federation	11 (1.0)	53 (2.1)	18 (1.4)	10 (1.3)	8 (1.1)
France	6 (0.7)	52 (1.5)	25 (1.2)	8 (0.8)	11 (0.9)
Italy	10 (1.0)	50 (1.8)	21 (1.6)	8 (1.0)	11 (1.2)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS-Advanced 2015

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and £.

(.) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.5: Description of the TIMSS Advanced 2015 High International Benchmark (550) of Advanced Mathematics Achievement**

550 High International Benchmark



**Summary**

*Students can apply a broad range of mathematical concepts and procedures in algebra, calculus, geometry, and trigonometry to analyze and solve multi-step problems set in routine and non-routine contexts.*

Students can analyze and solve algebra problems, including problems set in a practical context. They can solve problems requiring interpretation of information related to functions and graphs of functions. They can determine a sum of an arithmetic sequence and solve quadratic and other inequalities. They can simplify logarithmic expressions and multiply complex numbers.

In calculus, students have a basic understanding of continuity and differentiability. They can analyze equations of functions and graphs of functions. They can relate the graphs of functions to graphs and signs of their first and second derivatives. Students show some conceptual understanding of definite integrals.

Students can use trigonometric properties to solve a variety of problems involving trigonometric functions and geometric figures. They can use the Cartesian plane to solve problems, identify a vector perpendicular to a given vector, and prove that a quadrilateral given in the coordinate system is a parallelogram.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M2.5.1: High International Benchmark – Example Item 1**

Country	Percent Full Credit
‡ Lebanon	65 (3.3) ●
Russian Federation 6hr+	58 (3.1) ●
Russian Federation	41 (2.1) ●
Italy	36 (2.0)
Slovenia	34 (2.0)
International Avg.	33 (0.7)
† Portugal	31 (1.6)
‡ United States	26 (2.3) ▼
France	26 (1.4) ▼
Norway	22 (1.7) ▼
Sweden	18 (1.1) ▼

**Content Domain: Algebra**  
**Cognitive Domain: Applying**  
**Description: Determines the values of two constants in a rational expression given its graph with two specified points**

The graph of the function  $f(x) = \frac{ax+5}{x+b}$  is shown above. Find the values of  $a$  and  $b$ .

$a = \underline{\quad 5 \quad}$

$b = \underline{\quad -3 \quad}$

The answer shown illustrates the type of response that would receive full credit (1 point). To receive 1 point, students indicated  $a = 5$  and  $b = -3$ .

- Percent significantly higher than international average
- ▼ Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and †.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M2.5.1: High International Benchmark – Example Item 1  
(Continued)**

Scoring Guide		
Code	Response	Item: MA33179
<b>Correct Response</b>		
10	$a = 5$ $b = -3$	
<b>Incorrect Response</b>		
70	$a = 5$ correct only	
71	$b = -3$ correct only	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
<b>Nonresponse</b>		
NR	No Response	

Country	Percent of Students in Each Scoring Guide Category				
	Correct Student Response	Incorrect Student Responses			
	10	70	71	79	NR*
‡ Lebanon	65 (3.3)	16 (2.0)	0 (0.2)	12 (2.7)	7 (1.7)
Russian Federation 6hr+	58 (3.1)	12 (1.2)	0 (0.1)	14 (1.4)	16 (2.9)
Russian Federation	41 (2.1)	14 (1.4)	1 (0.2)	23 (1.7)	22 (1.6)
Italy	36 (2.0)	13 (1.2)	0 (0.2)	21 (1.7)	31 (2.2)
Slovenia	34 (2.0)	28 (2.5)	0 (0.2)	30 (1.5)	8 (1.2)
† Portugal	31 (1.6)	20 (1.4)	2 (0.4)	32 (1.7)	16 (1.3)
‡ United States	26 (2.3)	16 (1.4)	1 (0.4)	48 (2.1)	9 (1.2)
France	26 (1.4)	13 (0.9)	1 (0.4)	34 (1.5)	26 (1.5)
Norway	22 (1.7)	15 (1.0)	2 (0.4)	38 (2.0)	23 (1.3)
Sweden	18 (1.1)	12 (0.8)	2 (0.4)	48 (1.4)	21 (1.4)

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and §.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M2.5.2: High International Benchmark – Example Item 2**

Country	Percent Full Credit
Sweden	57 (1.4) ●
Russian Federation 6hr+	55 (3.0) ●
Norway	51 (2.0) ●
† Portugal	43 (1.7)
‡ United States	41 (2.4)
International Avg.	41 (0.7)
Russian Federation	39 (1.9)
‡ Lebanon	36 (2.7)
France	32 (1.6) ◐
Slovenia	32 (1.8) ◐
Italy	32 (1.9) ◐

**Content Domain: Algebra**  
**Cognitive Domain: Applying**  
**Description: Solves a word problem involving dimensions of two cylindrical containers given their volumes**

A manufacturer produces cylindrical cans with a diameter of 6 cm to hold 600 cm<sup>3</sup> of soup. He wants to change the diameter of the cans, leaving the height unchanged so that they will hold 750 cm<sup>3</sup> of soup. What will the new diameter be?

Show your work.

$$\begin{aligned}
 V_1 &= \pi r_1^2 h & V_2 &= \pi r_2^2 h \\
 \frac{V_1}{V_2} &= \frac{\pi r_1^2 h}{\pi r_2^2 h} \\
 r_2^2 &= \frac{V_2}{V_1} r_1^2 = \frac{750}{600} (3^2) \\
 &= \frac{5}{4} (3^2) \\
 &= \frac{3\sqrt{5}}{2} \\
 \text{New diameter} &= 3\sqrt{5} \text{ cm}
 \end{aligned}$$

The answer shown illustrates the type of response that would receive full credit (2 points). To receive 2 points, student work included a mathematical expression equating the ratio of the volumes of the cylinders and the ratio of the product of their squared radii and heights, substitution of the relevant values, and the final answer.

- Percent significantly higher than international average
- ◐ Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.5.2: High International Benchmark – Example Item 2  
(Continued)**

Code		Response	Item: MA23187
<b>Correct Response</b>			
20	Any of $3\sqrt{5}$ cm, 6.72 cm, 6.7 cm, or other equivalent with correct work		
21	An equation for finding the new diameter is correctly presented, followed by a statement about using the calculator to solve the equation, giving a correct answer		
<b>Partially Correct Response</b>			
10	Correct method but numerical error		
11	A correct equation for finding the new diameter is given but there is a subsequent error		
12	Code 10 or code 11, but using calculator		
<b>Incorrect Response</b>			
70	Calculator used—answer incorrect or explanation inadequate		
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)		
<b>Nonresponse</b>			
NR	No Response		

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

Country	Percent of Students in Each Scoring Guide Category							
	Correct Student Responses					Incorrect Student Responses		
	20	21	10	11	12	70	79	NR*
Sweden	57 (1.4)	0 (0.1)	8 (1.0)	1 (0.2)	0 (0.0)	0 (0.0)	25 (1.0)	8 (0.8)
Russian Federation 6hr+	54 (3.0)	1 (0.4)	12 (1.4)	0 (0.2)	0 (0.1)	0 (0.1)	20 (1.9)	12 (1.8)
Norway	51 (2.0)	0 (0.2)	9 (1.0)	15 (1.3)	0 (0.0)	0 (0.0)	17 (1.7)	8 (1.0)
† Portugal	43 (1.7)	0 (0.1)	5 (0.7)	9 (0.9)	0 (0.0)	0 (0.1)	35 (1.6)	8 (0.9)
‡ United States	40 (2.5)	1 (0.4)	12 (1.8)	6 (1.0)	0 (0.2)	0 (0.1)	36 (2.7)	4 (1.0)
Russian Federation	39 (1.9)	1 (0.4)	11 (1.3)	1 (0.3)	0 (0.1)	0 (0.1)	31 (1.5)	17 (1.5)
‡ Lebanon	36 (2.7)	0 (0.0)	11 (1.9)	2 (0.7)	0 (0.0)	0 (0.0)	32 (2.6)	19 (2.1)
France	32 (1.6)	0 (0.0)	10 (0.8)	2 (0.4)	0 (0.1)	0 (0.2)	42 (1.6)	14 (1.2)
Slovenia	32 (1.8)	0 (0.0)	16 (1.9)	17 (1.5)	0 (0.1)	0 (0.0)	27 (2.0)	7 (0.8)
Italy	32 (1.9)	0 (0.0)	3 (0.7)	6 (0.8)	0 (0.0)	0 (0.1)	35 (2.0)	24 (1.8)

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

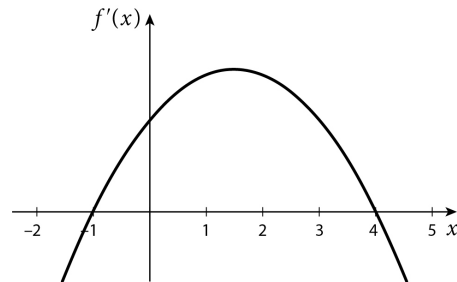


**Exhibit M2.5.3: High International Benchmark – Example Item 3**

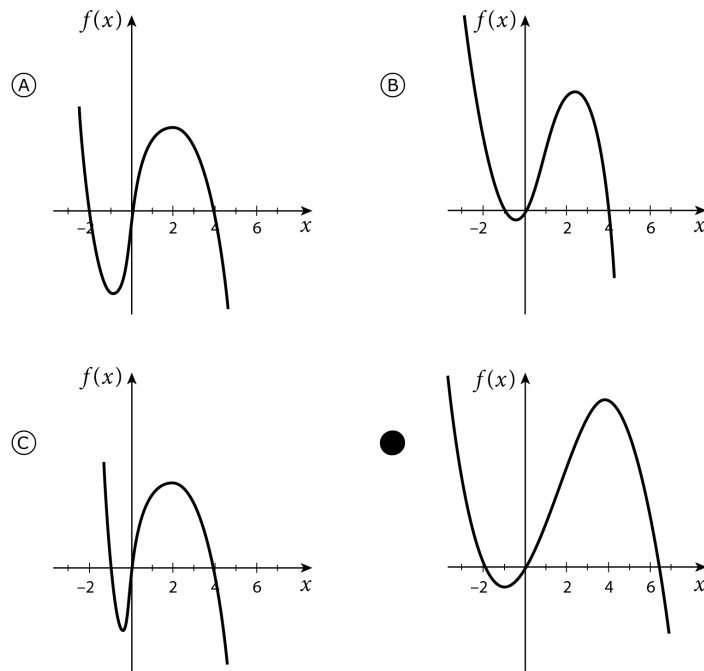
Country	Percent Correct
Russian Federation 6hr+	69 (2.8) ●
‡ Lebanon	60 (2.8) ●
‡ United States	58 (2.8) ●
Russian Federation	58 (2.4) ●
† Portugal	54 (1.7) ●
Norway	53 (2.3)
France	50 (1.6)
<b>International Avg.</b>	<b>50 (0.7)</b>
Sweden	49 (1.5)
Slovenia	34 (2.1) ▼
Italy	32 (1.7) ▼

**Content Domain: Calculus**  
**Cognitive Domain: Reasoning**  
**Description: Identifies the graph of a function given the graph of its first derivative**

The graph of the first derivative of the function  $f$  is shown below.



Which graph best represents the function  $f$ ?



- Percent significantly higher than international average
- ▼ Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and †.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M2.5.3: High International Benchmark – Example Item 3  
(Continued)**

Country	Percent of Students Responding to Each Answer Option				
	A	B	C	D	NR*
Russian Federation 6hr+	7 (1.2)	13 (1.9)	8 (1.0)	69 (2.8)	4 (0.8)
‡ Lebanon	7 (1.5)	15 (2.1)	11 (1.7)	60 (2.8)	8 (1.6)
‡ United States	9 (1.0)	21 (1.6)	7 (1.1)	58 (2.8)	5 (2.0)
Russian Federation	8 (1.0)	17 (1.2)	14 (1.3)	58 (2.4)	4 (0.6)
† Portugal	7 (1.0)	23 (1.5)	11 (1.0)	54 (1.7)	5 (0.6)
Norway	11 (1.2)	21 (2.1)	9 (1.1)	53 (2.3)	7 (0.8)
France	9 (0.9)	23 (1.2)	13 (1.0)	50 (1.6)	5 (0.7)
Sweden	11 (1.1)	22 (1.1)	11 (0.8)	49 (1.5)	7 (0.7)
Slovenia	13 (1.2)	31 (1.5)	17 (1.6)	34 (2.1)	5 (1.0)
Italy	10 (1.1)	28 (1.6)	17 (1.3)	32 (1.7)	13 (1.2)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS-Advanced 2015

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

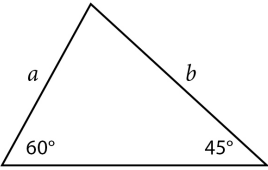
See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.5.4: High International Benchmark – Example Item 4**

Country	Percent Correct	Content Domain: Geometry
		Cognitive Domain: Applying
		Description: Determines the ratio of the squares of two sides of a scalene triangle given two of its angles

Russian Federation 6hr+	69 (2.5) ●
‡ Lebanon	62 (3.4) ●
Russian Federation	59 (1.5) ●
‡ United States	43 (2.7)
Italy	42 (2.0)
International Avg.	42 (0.7)
Slovenia	38 (1.5) ▼
Norway	35 (1.6) ▼
Sweden	34 (1.4) ▼
† Portugal	33 (1.4) ▼
France	29 (1.4) ▼



What is the value of  $\frac{a^2}{b^2}$ ?

$\frac{2}{3}$   
 (B)  $\frac{3}{2}$   
 (C)  $\frac{2}{\sqrt{3}}$   
 (D)  $\frac{\sqrt{2}}{\sqrt{3}}$

- Percent significantly higher than international average
- ▼ Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and †.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M2.5.4: High International Benchmark – Example Item 4  
(Continued)**

Country	Percent of Students Responding to Each Answer Option				
	A	B	C	D	NR*
Russian Federation 6hr+	69 (2.5)	12 (1.4)	9 (1.3)	6 (0.9)	4 (0.9)
‡ Lebanon	62 (3.4)	12 (1.9)	8 (1.6)	4 (1.2)	14 (2.2)
Russian Federation	59 (1.5)	16 (1.0)	11 (0.9)	10 (1.0)	4 (0.7)
‡ United States	43 (2.7)	14 (1.3)	20 (1.9)	12 (1.6)	11 (2.5)
Italy	42 (2.0)	18 (1.3)	14 (1.5)	12 (1.2)	14 (1.2)
Slovenia	38 (1.5)	21 (1.5)	18 (1.4)	13 (1.2)	11 (1.0)
Norway	35 (1.6)	13 (1.3)	21 (1.7)	17 (1.2)	14 (1.0)
Sweden	34 (1.4)	14 (1.0)	21 (1.3)	19 (1.3)	12 (1.0)
† Portugal	33 (1.4)	24 (1.2)	17 (1.4)	10 (1.2)	16 (1.1)
France	29 (1.4)	19 (1.2)	20 (1.3)	12 (0.9)	20 (1.3)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS-Advanced 2015

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and £.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.5.5: High International Benchmark – Example Item 5**

Country	Percent Full Credit
Russian Federation 6hr+	52 (3.3) ●
‡ Lebanon	45 (3.2) ●
‡ United States	36 (2.6) ●
Russian Federation	32 (1.9) ●
† Portugal	30 (1.8)
Norway	28 (2.4)
International Avg.	27 (0.7)
Slovenia	20 (1.7) ▼
Sweden	18 (1.0) ▼
Italy	17 (1.6) ▼
France	13 (1.0) ▼

**Content Domain: Geometry**  
**Cognitive Domain: Reasoning**  
**Description: Finds the maximum value of a trigonometric function and a value of the independent variable at which it occurs**

The number of animals in a certain population  $P(t)$  varies periodically with time  $t$ . This can be modeled by

$$P(t) = 900 + 600 \sin\left(t + \frac{\pi}{3}\right)$$

What is the maximum number of animals?  
Indicate one of the times at which the maximum occurs.  
Maximum number of animals:  
 $P(t) = \underline{1500}$   
One time at which maximum occurs:  
 $t = \underline{\frac{\pi}{6}}$

The answer shown illustrates the type of response that would receive full credit (2 points). To receive 2 points, students indicated both that the maximum number of animals is 1500 and that the time at which the maximum occurs is  $\pi/6$  (or equivalent).

- Percent significantly higher than international average
- ▼ Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and †.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M2.5.5: High International Benchmark – Example Item 5  
(Continued)**

Scoring Guide		
<b>Code</b>	<b>Response</b>	<b>Item: MA33232</b>
<b>Correct Response</b>		
20	$P(t) = 1500$ $t = \frac{\pi}{6}$ (or any other value of the type $\frac{\pi}{6} + 2k\pi$ )	
<b>Partially Correct Response</b>		
10	$P(t)$ correct only	
11	$t$ correct only	
<b>Incorrect Response</b>		
79	Incorrect (including crossed out, erased, stray marks, illegible, or off task)	
<b>Nonresponse</b>		
NR	No Response	

Country	Percent of Students in Each Scoring Guide Category				
	Correct Student Responses			Incorrect Student Responses	
	20	10	11	79	NR*
Russian Federation 6hr+	52 (3.3)	8 (1.2)	2 (0.5)	11 (1.2)	28 (3.1)
‡ Lebanon	45 (3.2)	9 (1.7)	4 (2.1)	17 (2.5)	26 (2.2)
‡ United States	36 (2.6)	10 (1.3)	6 (1.2)	32 (2.5)	16 (1.7)
Russian Federation	32 (1.9)	11 (1.1)	2 (0.8)	14 (1.2)	41 (1.7)
† Portugal	30 (1.8)	12 (1.2)	3 (1.1)	26 (1.5)	30 (1.8)
Norway	28 (2.4)	21 (1.2)	2 (0.8)	23 (1.5)	27 (1.8)
Slovenia	20 (1.7)	9 (1.0)	2 (0.5)	28 (1.7)	40 (2.0)
Sweden	18 (1.0)	32 (1.6)	1 (0.2)	26 (1.1)	23 (1.5)
Italy	17 (1.6)	4 (0.6)	2 (0.6)	16 (1.5)	60 (2.1)
France	13 (1.0)	22 (1.1)	0 (0.2)	25 (1.5)	39 (1.5)

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and †.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015



**Exhibit M2.6: Description of the TIMSS Advanced 2015 Advanced International Benchmark (625) of Advanced Mathematics Achievement****625 Advanced International Benchmark****Summary**











*Students demonstrate thorough understanding of concepts, mastery of procedures, and mathematical reasoning skills. They can solve problems in complex contexts in algebra, calculus, geometry, and trigonometry.*

In algebra, students can reason with functions to solve pure mathematical problems. They demonstrate facility with complex numbers and permutations and can find sums of algebraic and infinite geometric series.



In calculus, students demonstrate thorough understanding of continuity and differentiability. They can solve problems about optimization in different contexts and justify their solutions. They can use definite integrals to calculate the area between two curves.

Students use geometric reasoning to solve complex problems. They use properties of vectors to express relationships among vectors. They can use trigonometric properties including the sine and cosine rules to solve non-routine problems about geometric figures.

**Exhibit M2.6.1: Advanced International Benchmark – Example Item 1**

Country	Percent Full Credit	Content Domain: Algebra
		Cognitive Domain: Applying
		Description: Determines the intersection of two functions in terms of an unknown, non-zero coefficient
Russian Federation 6hr+	50 (2.8) 	<p>Let <math>a</math> be a non-zero constant. Find the two <math>x</math>-values where the graphs of <math>y = 10^6 ax</math> and <math>y = \frac{x^2}{10^6}</math> intersect.</p> <p>Answer: <math>x = 0, x = 10^{12} a</math></p>
‡ Lebanon	39 (2.6) 	
Russian Federation	35 (1.9) 	
Italy	29 (1.6) 	
Slovenia	26 (1.7) 	
International Avg.	20 (0.5)	
† Portugal	13 (1.1) 	
Norway	11 (1.2) 	
Sweden	9 (1.1) 	
France	8 (1.0) 	
‡ United States	7 (1.0) 	

The answer shown illustrates the type of response that would receive full credit (2 points). To receive 2 points, students indicated that  $x = 0$  and  $x = 10^{12} a$ .

-  Percent significantly higher than international average
-  Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and †.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015



**Exhibit M2.6.1: Advanced International Benchmark – Example Item 1 (Continued)**

Scoring Guide		
<b>Code</b>	<b>Response</b>	<b>Item: MA33121</b>
<b>Correct Response</b>		
<b>20</b>	$x = 0$ and $x = 10^{12}a$	
<b>Partially Correct Response</b>		
<b>10</b>	$x = 0$ correct only	
<b>11</b>	$x = 10^{12}a$ correct only	
<b>Incorrect Response</b>		
<b>79</b>	Incorrect (including crossed out, erased, stray marks, illegible, or off task)	
<b>Nonresponse</b>		
<b>NR</b>	No Response	

Country	Percent of Students in Each Scoring Guide Category				
	Correct Student Responses			Incorrect Student Responses	
	20	10	11	79	NR*
Russian Federation 6hr+	50 (2.8)	11 (1.4)	4 (0.8)	14 (2.9)	21 (2.8)
‡ Lebanon	39 (2.6)	9 (1.7)	22 (2.7)	19 (2.8)	11 (2.2)
Russian Federation	35 (1.9)	13 (1.6)	3 (0.4)	15 (1.5)	33 (1.7)
Italy	29 (1.6)	8 (0.8)	5 (0.7)	15 (1.4)	43 (2.3)
Slovenia	26 (1.7)	9 (0.9)	17 (1.7)	37 (2.1)	11 (1.2)
† Portugal	13 (1.1)	12 (0.9)	15 (1.1)	28 (1.4)	32 (1.9)
Norway	11 (1.2)	15 (1.6)	11 (1.4)	26 (2.2)	38 (2.5)
Sweden	9 (1.1)	10 (0.7)	10 (0.9)	34 (1.5)	37 (1.5)
France	8 (1.0)	11 (0.7)	11 (1.0)	29 (1.4)	41 (1.3)
‡ United States	7 (1.0)	22 (1.5)	16 (1.6)	42 (2.1)	13 (1.3)

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and †.

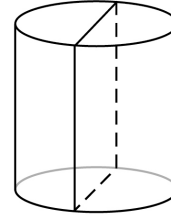
( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M2.6.2: Advanced International Benchmark – Example Item 2**

Country	Percent Correct
Russian Federation 6hr+	52 (2.4) ●
Russian Federation	45 (1.7) ●
Slovenia	35 (1.8) ●
‡ United States	31 (1.9)
Sweden	31 (1.2)
International Avg.	30 (0.6)
Italy	29 (1.8)
† Portugal	28 (1.6)
Norway	28 (1.6)
France	25 (1.3) ▼
‡ Lebanon	19 (1.7) ▼

Content Domain: Calculus  
Cognitive Domain: Applying  
Description: Maximizes the volume of a cylinder given a relationship between its height and diameter



The intersection of a cylinder with a plane through its axis is a rectangle of perimeter 6 m. The radius of the cylinder satisfying this condition and having maximum volume is

- (A) 2.5 m
- (B) 2 m
- (C) 1.5 m
- 1 m
- (E) 0.5 m

- Percent significantly higher than international average
- ▼ Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.6.2: Advanced International Benchmark – Example Item 2  
(Continued)**

Country	Percent of Students Responding to Each Answer Option					
	A	B	C	D	E	NR*
Russian Federation 6hr+	9 (1.0)	14 (1.8)	9 (1.5)	52 (2.4)	13 (2.1)	3 (0.5)
Russian Federation	8 (0.9)	14 (1.1)	14 (1.4)	45 (1.7)	15 (1.2)	3 (0.5)
Slovenia	9 (1.1)	14 (1.0)	15 (1.2)	35 (1.8)	20 (2.0)	6 (0.9)
‡ United States	10 (1.5)	15 (1.5)	20 (1.5)	31 (1.9)	21 (2.2)	3 (0.8)
Sweden	10 (1.1)	20 (1.1)	15 (1.2)	31 (1.2)	13 (1.4)	10 (0.9)
Italy	7 (1.0)	15 (1.3)	16 (1.1)	29 (1.8)	17 (1.3)	16 (1.3)
† Portugal	6 (1.0)	12 (1.1)	15 (1.2)	28 (1.6)	29 (1.4)	9 (0.9)
Norway	10 (1.0)	18 (1.5)	16 (1.4)	28 (1.6)	19 (1.4)	9 (1.1)
France	13 (1.0)	22 (1.0)	17 (1.2)	25 (1.3)	14 (1.0)	10 (0.8)
‡ Lebanon	15 (1.9)	13 (2.0)	17 (2.1)	19 (1.7)	9 (1.5)	27 (2.9)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

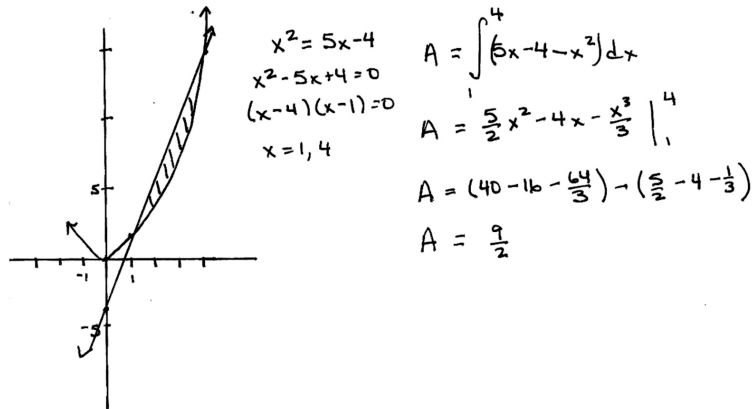
( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.6.3: Advanced International Benchmark – Example Item 3**

Country	Percent Full Credit
‡ United States	37 (2.8) ●
Sweden	23 (1.3) ●
Russian Federation 6hr+	22 (2.2) ●
‡ Lebanon	20 (2.8)
Norway	20 (2.3)
International Avg.	16 (0.6)
Russian Federation	16 (1.5)
Slovenia	15 (1.2)
France	7 (0.8) ▼
Italy	6 (1.0) ▼
† Portugal	1 (0.3) ▼

Content Domain: Calculus  
 Cognitive Domain: Applying  
 Description: Calculates the area between the graphs of a linear and a quadratic function

What is the area enclosed by the graphs of functions  $y = x^2$  and  $y = 5x - 4$ ?  
 Show your work.



The answer shown illustrates the type of response that would receive full credit (2 points). To receive 2 points, student work showed the subtraction of the definite integrals from  $x = 1$  to  $x = 4$  of  $5x - 4$  and  $x^2$ , respectively, and the final answer.

- Percent significantly higher than international average
- ▼ Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and §.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M2.6.3: Advanced International Benchmark – Example Item 3  
(Continued)**

Scoring Guide		
Code	Response	Item: MA23043
<b>Correct Response</b>		
20	Integrations and subtraction shown correctly to give area of $4\frac{1}{2}$ , $\frac{9}{2}$ , 4.5, or equivalent <i>Note:</i> No need to explicitly show how points of intersection of the two functions was determined	
21	Correct solution by use of calculator. Calculator use is described in examples	
<b>Partially Correct Response</b>		
10	Correct method with computation error	
11	Correct method using calculator but correct answer not given	
12	$-4\frac{1}{2}$ or equivalent with correct method shown	
<b>Incorrect Response</b>		
70	Calculator used—answer incorrect or explanation inadequate	
79	Other incorrect (including crossed out, erased, stray marks, illegible, or off task)	
<b>Nonresponse</b>		
NR	No Response	

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

Country	Percent of Students in Each Scoring Guide Category							
	Correct Student Responses					Incorrect Student Responses		
	20	21	10	11	12	70	79	NR*
‡ United States	24 (2.1)	14 (1.6)	13 (1.3)	2 (0.5)	1 (0.3)	1 (0.3)	35 (2.5)	11 (1.7)
Sweden	16 (1.1)	6 (0.7)	6 (0.8)	0 (0.3)	1 (0.2)	3 (0.6)	36 (1.4)	32 (1.6)
Russian Federation 6hr+	22 (2.2)	0 (0.1)	8 (1.1)	0 (0.1)	1 (0.4)	0 (0.1)	38 (2.2)	32 (3.1)
‡ Lebanon	20 (2.6)	1 (0.7)	15 (2.0)	0 (0.0)	1 (0.5)	0 (0.0)	43 (2.8)	20 (3.2)
Norway	16 (2.0)	4 (0.9)	10 (1.2)	0 (0.0)	1 (0.5)	2 (0.5)	29 (1.5)	38 (2.4)
Russian Federation	16 (1.5)	0 (0.1)	5 (0.8)	0 (0.0)	0 (0.2)	0 (0.0)	34 (1.9)	45 (2.0)
Slovenia	15 (1.2)	0 (0.1)	15 (1.9)	0 (0.1)	2 (0.5)	0 (0.1)	55 (2.0)	13 (1.5)
France	5 (0.8)	1 (0.3)	2 (0.5)	0 (0.2)	1 (0.4)	2 (0.4)	40 (1.7)	47 (1.6)
Italy	6 (1.0)	0 (0.1)	8 (1.0)	0 (0.0)	1 (0.2)	0 (0.0)	32 (1.7)	54 (2.0)
† Portugal	0 (0.1)	1 (0.3)	0 (0.0)	0 (0.0)	0 (0.0)	0 (0.2)	48 (1.8)	51 (1.8)

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and †.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.



**Exhibit M2.6.4: Advanced International Benchmark – Example Item 4**

Country	Percent Correct	Content Domain: Geometry
		Cognitive Domain: Knowing
		Description: Uses properties of vectors to analyze equivalence of conditions involving the sum and difference of two vectors
Russian Federation 6hr+	56 (2.7) ●	<p>If <math>\vec{a} \neq \vec{0}</math> and <math>\vec{b} \neq \vec{0}</math>, which of the following is equivalent to the equation <math> \vec{a} + \vec{b}  =  \vec{a} - \vec{b} </math>?</p> <p>(A) <math> \vec{a}  =  \vec{b} </math></p> <p>(B) <math>\vec{a}</math> and <math>\vec{b}</math> are parallel vectors.</p> <p>● <math>\vec{a}</math> and <math>\vec{b}</math> are perpendicular vectors.</p> <p>(D) <math>\vec{a} + \vec{b} = \vec{0}</math></p>
Russian Federation	45 (1.9) ●	
Slovenia	43 (2.2) ●	
Sweden	41 (1.6)	
Norway	40 (2.2)	
† Portugal	39 (2.0)	
‡ United States	39 (2.1)	
International Avg.	39 (0.7)	
France	34 (1.6) ▼	
Italy	34 (2.0) ▼	
‡ Lebanon	32 (2.2) ▼	

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

- Percent significantly higher than international average
- ▼ Percent significantly lower than international average

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M2.6.4: Advanced International Benchmark – Example Item 4 (Continued)**

Country	Percent of Students Responding to Each Answer Option				
	A	B	C	D	NR*
Russian Federation 6hr+	9 (1.4)	21 (2.0)	56 (2.7)	9 (1.0)	5 (1.0)
Russian Federation	11 (1.0)	27 (1.7)	45 (1.9)	11 (1.2)	5 (0.7)
Slovenia	10 (1.0)	27 (1.5)	43 (2.2)	14 (1.2)	6 (0.8)
Sweden	11 (0.9)	30 (1.3)	41 (1.6)	10 (0.9)	8 (0.7)
Norway	10 (0.9)	31 (1.8)	40 (2.2)	13 (1.2)	6 (1.0)
† Portugal	11 (1.2)	28 (1.6)	39 (2.0)	13 (1.2)	9 (0.9)
‡ United States	9 (1.1)	36 (2.2)	39 (2.1)	9 (1.2)	7 (2.2)
France	14 (1.0)	28 (1.6)	34 (1.6)	15 (1.0)	9 (0.9)
Italy	14 (1.3)	30 (1.7)	34 (2.0)	9 (1.0)	13 (1.2)
‡ Lebanon	13 (2.1)	28 (2.9)	32 (2.2)	13 (2.1)	14 (2.0)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS-Advanced 2015

\* No Response.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

# CHAPTER M3: ACHIEVEMENT IN CONTENT AND COGNITIVE DOMAINS

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS



**IEA**

**TIMSS & PIRLS**  
International Study Center  
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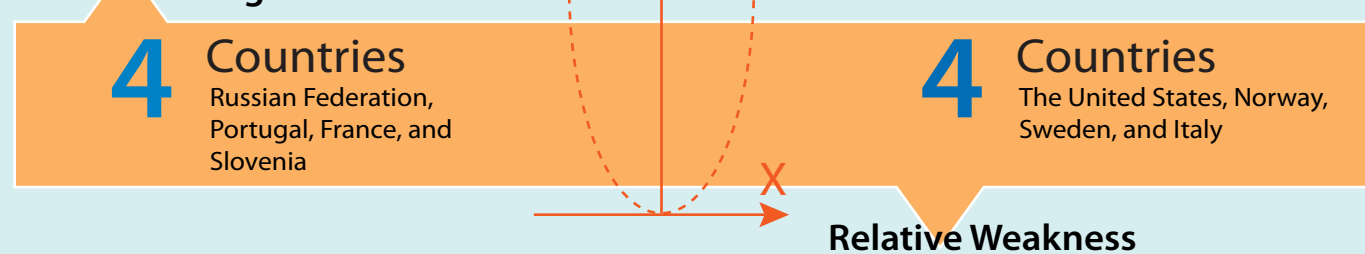


### Achievement by Content Domains

Within mathematics, TIMSS Advanced provided results for three content domains—Algebra, Calculus, and Geometry. Each country demonstrated strengths in one or two content domains compared to mathematics achievement overall, and weaknesses in one or two content domains.

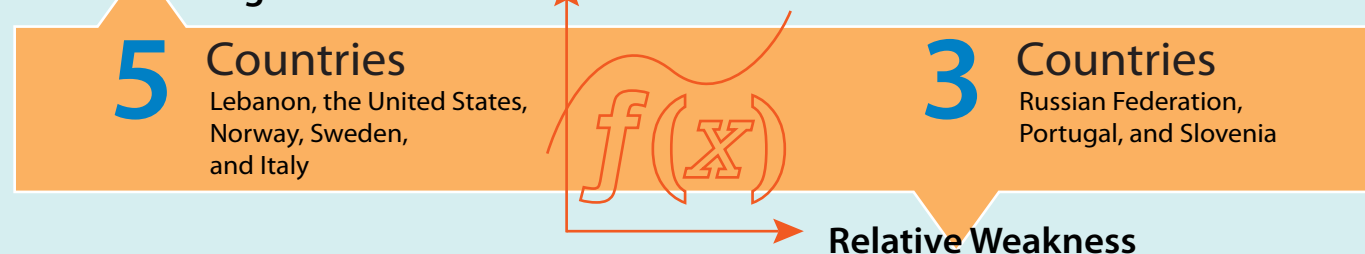
#### Algebra

Relative Strength



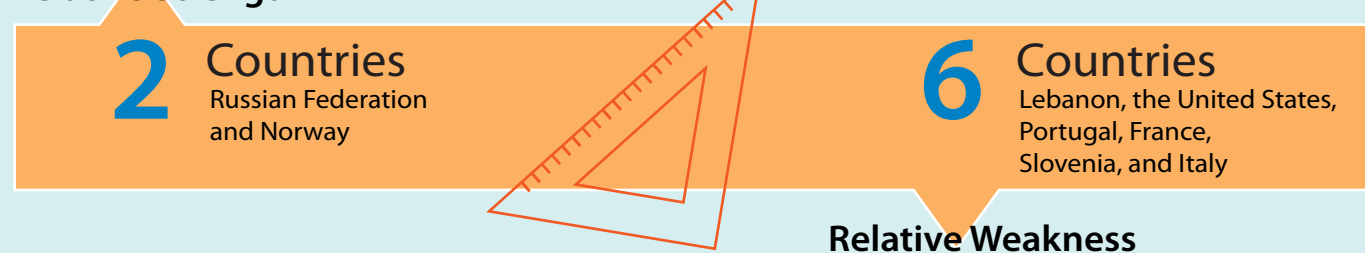
#### Calculus

Relative Strength



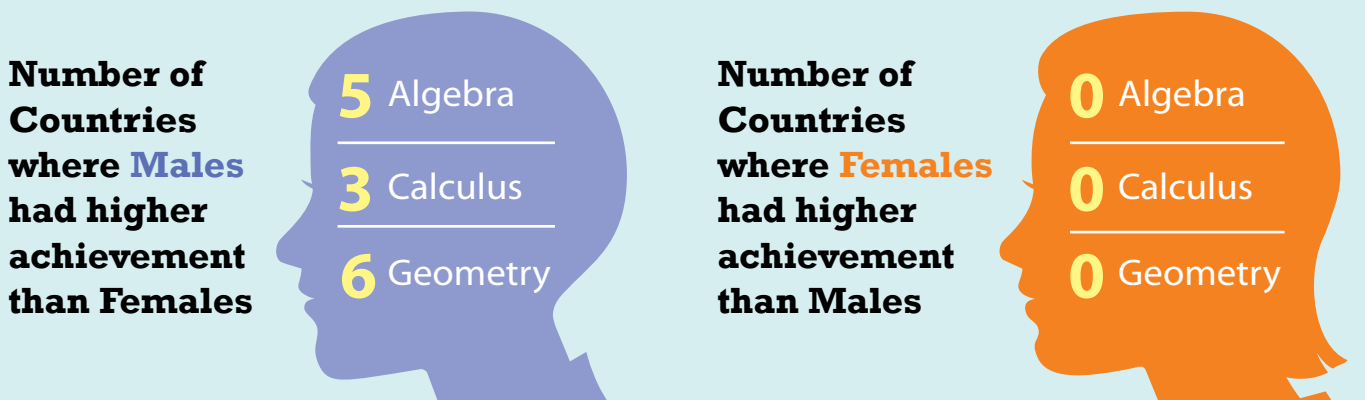
#### Geometry

Relative Strength



#### Differences reflected in Achievement by Gender

Achievement differences in content domains by gender reflect males' higher achievement in 6 countries in mathematics overall.



### Achievement by Cognitive Domains

TIMSS Advanced provided results for three cognitive domains—Knowing, Applying, and Reasoning. Although there was some balance in achievement across cognitive domains, most countries had at least one strength and one weakness compared to mathematics achievement overall.

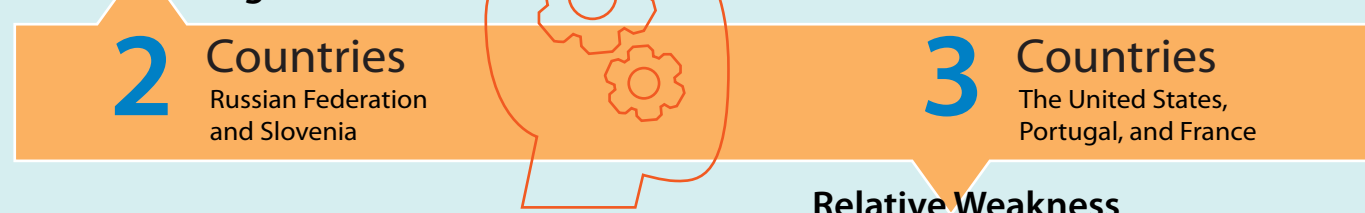
#### Knowing

Relative Strength



#### Applying

Relative Strength



#### Reasoning

Relative Strength



#### Differences reflected in Achievement by Gender

Besides reflecting males' higher achievement in 6 countries in mathematics overall, achievement differences in the cognitive domains by gender show a male advantage, especially in Reasoning.





**Exhibit M3.1: Achievement in Advanced Mathematics Content Domains**

Country	Overall Advanced Mathematics Average Scale Score	Algebra (37 items)		Calculus (34 items)		Geometry (30 items)	
		Average Scale Score	Difference from Overall Advanced Mathematics Score	Average Scale Score	Difference from Overall Advanced Mathematics Score	Average Scale Score	Difference from Overall Advanced Mathematics Score
Russian Federation 6hr+	540 (7.8)	556 (9.0)	16 (3.9) ▲	513 (8.0)	-27 (2.3) ▼	560 (8.4)	20 (3.2) ▲
‡ Lebanon	532 (3.1)	525 (4.0)	-6 (3.6)	544 (3.9)	12 (2.8) ▲	526 (3.7)	-6 (2.3) ▼
‡ United States	485 (5.2)	478 (5.0)	-7 (1.7) ▼	504 (6.0)	19 (2.9) ▲	455 (5.7)	-30 (2.6) ▼
Russian Federation	485 (5.7)	495 (6.3)	10 (1.9) ▲	459 (5.9)	-26 (1.2) ▼	500 (5.8)	15 (1.0) ▲
† Portugal	482 (2.5)	495 (2.7)	12 (1.5) ▲	476 (2.6)	-6 (1.4) ▼	464 (3.2)	-18 (1.5) ▼
France	463 (3.1)	469 (2.9)	7 (1.8) ▲	466 (3.2)	3 (1.8)	441 (3.7)	-22 (1.3) ▼
Slovenia	460 (3.4)	474 (3.5)	14 (1.1) ▲	437 (4.4)	-23 (2.0) ▼	456 (4.0)	-4 (1.4) ▼
Norway	459 (4.6)	446 (4.1)	-13 (1.6) ▼	463 (5.3)	4 (1.5) ▲	473 (4.6)	14 (2.0) ▲
Sweden	431 (4.0)	422 (4.1)	-9 (1.2) ▼	438 (3.9)	7 (1.5) ▲	430 (3.7)	-1 (1.4)
Italy	422 (5.3)	414 (5.1)	-8 (2.2) ▼	433 (5.2)	11 (2.7) ▲	413 (5.7)	-9 (3.2) ▼

- ▲ Subscale score significantly higher than overall mathematics score
- ▼ Subscale score significantly lower than overall mathematics score

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and £.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015



**Exhibit M3.2: Achievement in Advanced Mathematics Content Domains by Gender**

Country	Algebra		Calculus		Geometry	
	Females	Males	Females	Males	Females	Males
Russian Federation 6hr+	544 (10.3)	567 (8.5) ▲	504 (9.3)	521 (7.6) ▲	548 (9.2)	571 (8.2) ▲
‡ Lebanon	525 (6.1)	526 (4.1)	548 (5.1)	542 (4.5)	523 (7.2)	527 (3.9)
‡ United States	466 (5.2)	490 (6.8) ▲	492 (6.4)	517 (7.6) ▲	435 (6.2)	474 (6.5) ▲
Russian Federation	489 (6.5)	501 (6.7) ▲	456 (6.3)	462 (6.3)	492 (6.1)	507 (6.2) ▲
† Portugal	494 (3.0)	495 (3.4)	477 (3.5)	474 (3.8)	457 (4.2)	471 (3.9) ▲
France	455 (3.4)	482 (3.5) ▲	453 (3.5)	478 (3.6) ▲	426 (4.2)	454 (3.8) ▲
Slovenia	464 (3.6)	487 (4.9) ▲	425 (4.8)	454 (5.7) ▲	441 (4.4)	478 (5.3) ▲
Norway	443 (4.5)	449 (5.1)	457 (6.1)	467 (6.0)	466 (5.4)	477 (6.0)
Sweden	413 (4.7)	428 (4.7) ▲	435 (5.0)	440 (4.7)	421 (4.4)	436 (4.3) ▲
Italy	420 (6.1)	411 (6.7)	439 (6.1)	429 (6.7)	413 (7.6)	413 (6.7)
International Avg.	463 (1.6)	474 (1.8) ▲	465 (1.8)	474 (1.9) ▲	453 (1.9)	471 (1.8) ▲

▲ Average significantly higher than other gender

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M3.3: Achievement in Advanced Mathematics Cognitive Domains**

Country	Overall Advanced Mathematics Average Scale Score	Knowing (32 items)		Applying (40 items)		Reasoning (29 items)	
		Average Scale Score	Difference from Overall Advanced Mathematics Score	Average Scale Score	Difference from Overall Advanced Mathematics Score	Average Scale Score	Difference from Overall Advanced Mathematics Score
Russian Federation 6hr+	540 (7.8)	538 (8.8)	-2 (2.0)	544 (8.1)	4 (2.0)	541 (7.2)	1 (2.1)
‡ Lebanon	532 (3.1)	543 (4.5)	11 (2.9) ▲	529 (3.8)	-3 (2.8)	527 (3.9)	-5 (2.2) ▼
‡ United States	485 (5.2)	488 (5.7)	3 (2.3)	480 (5.5)	-5 (2.0) ▼	484 (5.3)	-1 (2.2)
Russian Federation	485 (5.7)	478 (6.7)	-7 (1.7) ▼	491 (6.1)	6 (1.7) ▲	484 (5.3)	-1 (1.2)
† Portugal	482 (2.5)	479 (3.0)	-3 (1.6)	476 (2.9)	-6 (1.8) ▼	488 (3.5)	6 (2.2) ▲
France	463 (3.1)	475 (2.7)	13 (2.0) ▲	449 (3.4)	-14 (1.5) ▼	462 (3.1)	0 (0.9)
Slovenia	460 (3.4)	466 (3.5)	6 (1.7) ▲	465 (4.0)	5 (2.1) ▲	442 (4.0)	-17 (1.6) ▼
Norway	459 (4.6)	445 (4.1)	-14 (1.8) ▼	459 (5.1)	0 (2.0)	469 (4.4)	9 (1.4) ▲
Sweden	431 (4.0)	405 (4.7)	-26 (1.4) ▼	434 (3.6)	3 (1.5)	447 (3.9)	16 (2.0) ▲
Italy	422 (5.3)	423 (5.5)	1 (1.9)	425 (5.4)	3 (2.2)	411 (5.9)	-11 (3.1) ▼

- ▲ Subscale score significantly higher than overall mathematics score
- ▼ Subscale score significantly lower than overall mathematics score

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and §.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M3.4: Achievement in Advanced Mathematics Cognitive Domains by Gender**

Country	Knowing		Applying		Reasoning	
	Females	Males	Females	Males	Females	Males
Russian Federation 6hr+	527 (10.0)	547 (8.6) ▲	535 (9.4)	552 (7.7) ▲	530 (8.1)	550 (7.1) ▲
‡ Lebanon	540 (7.2)	544 (4.4)	534 (5.1)	527 (4.9)	525 (6.1)	527 (4.2)
‡ United States	474 (6.1)	502 (7.0) ▲	468 (5.7)	491 (6.6) ▲	468 (5.7)	501 (6.6) ▲
Russian Federation	474 (7.3)	482 (7.0)	487 (6.7)	495 (6.4)	478 (5.5)	490 (5.9) ▲
† Portugal	480 (3.4)	479 (3.5)	475 (3.4)	476 (3.3)	485 (4.0)	491 (3.9)
France	465 (2.9)	485 (3.3) ▲	434 (3.7)	463 (4.0) ▲	446 (3.6)	477 (3.7) ▲
Slovenia	456 (3.8)	481 (4.9) ▲	455 (3.3)	480 (6.8) ▲	428 (4.9)	464 (5.8) ▲
Norway	439 (5.3)	449 (5.2)	456 (6.4)	461 (5.7)	461 (4.9)	473 (5.2) ▲
Sweden	395 (5.7)	412 (5.3) ▲	429 (4.5)	437 (4.9)	438 (5.5)	453 (4.0) ▲
Italy	431 (5.9)	418 (7.0)	431 (6.3)	422 (6.6)	410 (7.2)	412 (7.4)
International Avg.	461 (1.8)	472 (1.8) ▲	463 (1.7)	472 (1.9) ▲	460 (1.8)	476 (1.8) ▲

▲ Average significantly higher than other gender

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

See Appendix MC.5 for sampling guidelines and sampling participation notes †, ‡, and ‡.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

# CHAPTER M4: HOME ENVIRONMENT AND FUTURE PLANS

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS



**IEA**

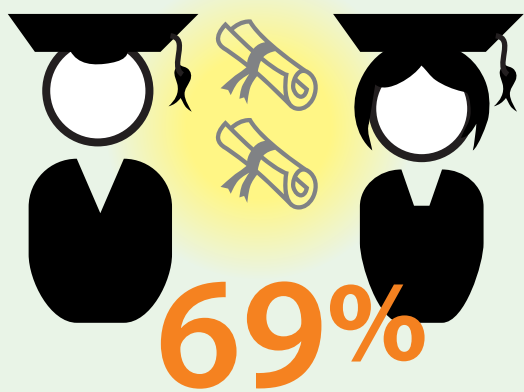
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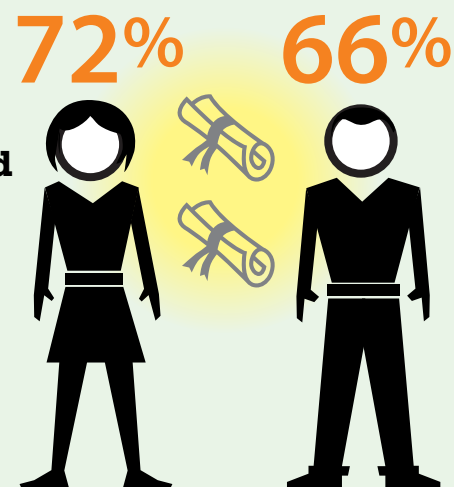
## Students' Plans for Future Study

Nearly all the Advanced Mathematics students planned to continue their education after finishing secondary school and the vast majority intend to obtain advanced degrees.

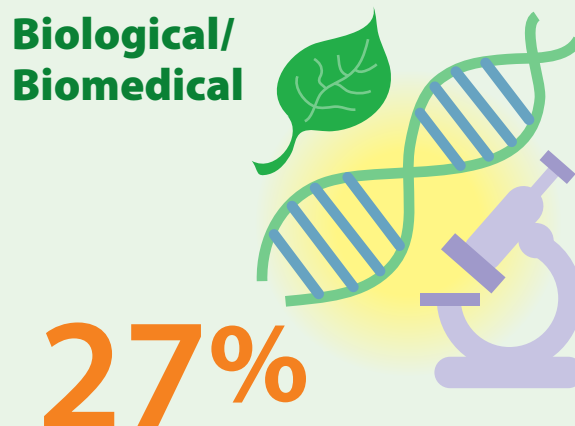
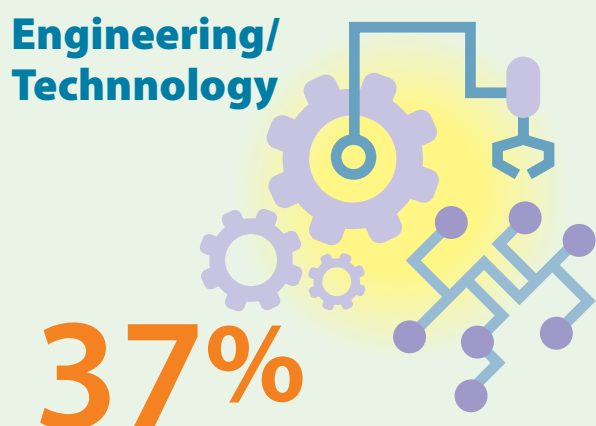


of students expect to obtain an advanced degree

Higher percentages of the Females taking advanced mathematics than of the Males expect to obtain an advanced degree.



The most popular areas of future study included:



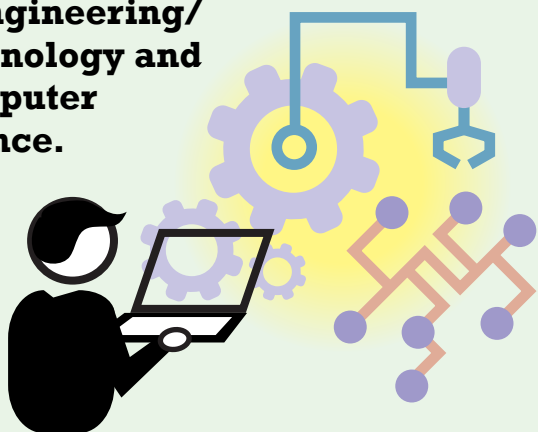
Education trailed behind (8%) as a future area of study.

## Students' Plans for Future Professions

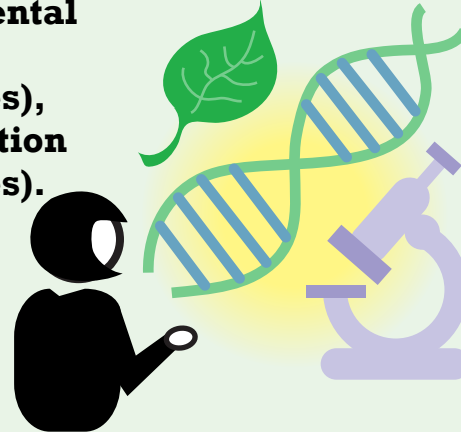
Students who studied advanced mathematics in secondary school reported considering a number of professions.

**A career in Engineering/Technology was the most popular choice, being considered by 60% on average. More than 40% on average also considered Computer Science, Biological/Biomedical, Education, and Finance.**

In every country, more Males were considering the professions of Engineering/Technology and Computer Science.



More Females were considering Biological/Biomedical (7 countries), Environmental Science (5 countries), and Education (6 countries).





**Exhibit M4.1: Home Educational Resources**

*Reported by Advanced Mathematics Students*

Students were scored according to their responses concerning the availability of four home resources on the *Home Educational Resources* scale. Students with **Many Resources** had a score of at least 11.6, which is the point on the scale corresponding to students reporting that they had more than 100 books in the home, both of the home study supports, and that at least one parent had finished university and that at least one parent was a professional, on average. Students with **Few Resources** had a score no higher than 6.0, which is the scale point corresponding to students reporting that they had 25 or fewer books in the home, neither of the home study supports, and that neither parent had gone beyond upper-secondary education and that neither parent was a small business owner or had a clerical or professional occupation, on average. All other students were assigned to the **Some Resources** category.

Country	Many Resources		Some Resources		Few Resources		Average Scale Score
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	
Norway	49 (2.3)	476 (4.1)	50 (2.1)	444 (4.9)	1 (0.2)	~ ~	11.2 (0.10)
Sweden	40 (1.5)	464 (4.7)	58 (1.4)	410 (4.4)	1 (0.2)	~ ~	10.7 (0.07)
United States	35 (1.9)	511 (6.5)	62 (1.8)	474 (5.4)	2 (0.4)	~ ~	10.5 (0.09)
Russian Federation 6hr+	30 (1.6)	567 (6.7)	70 (1.6)	529 (9.4)	0 (0.1)	~ ~	10.6 (0.06)
France	28 (1.1)	493 (3.7)	71 (1.1)	452 (3.2)	1 (0.3)	~ ~	10.2 (0.05)
Slovenia	23 (1.1)	489 (6.1)	77 (1.1)	452 (3.6)	0 (0.1)	~ ~	10.1 (0.06)
Portugal	20 (1.1)	513 (3.5)	73 (1.1)	476 (2.6)	7 (0.5)	461 (6.1)	9.3 (0.06)
Russian Federation	20 (1.0)	514 (5.2)	80 (1.0)	477 (6.2)	0 (0.1)	~ ~	10.1 (0.04)
Italy	16 (1.0)	471 (8.3)	79 (1.0)	416 (5.5)	5 (0.6)	347 (17.2)	9.3 (0.05)
Lebanon	8 (0.8)	546 (13.9)	82 (1.2)	535 (3.1)	10 (0.9)	496 (6.2)	8.7 (0.05)
International Avg.	27 (0.5)	497 (2.3)	70 (0.5)	460 (1.5)	3 (0.1)	435 (6.4)	

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

**Number of books in the home:**

- 1) 0-10
- 2) 11-25
- 3) 26-100
- 4) 101-200
- 5) More than 200

**Number of home study supports:**

- 1) None
- 2) Study desk/table or own room
- 3) Both

**Highest level of education of either parent:**

- 1) Finished some primary or lower secondary or did not go to school
- 2) Finished lower secondary
- 3) Finished upper secondary
- 4) Finished post-secondary education
- 5) Finished university or higher

**Highest level of occupation of either parent:**

- 1) Has never worked outside home for pay, general laborer, or semi-professional (skilled agricultural or fishery worker, craft or trade worker, plant or machine operator)
- 2) Clerical (clerk or service or sales worker)
- 3) Small business owner
- 4) Professional (corporate manager or senior official, professional, or technician or associate professional)

The diagram shows a horizontal scale with three points marked: 'Many Resources' at 11.6, 'Some Resources' in the middle, and 'Few Resources' at 6.0. Arrows indicate the direction of the scale.

**Exhibit M4.2: Students Working at a Paid Job on a Regular Basis During the School Year**

Reported by Advanced Mathematics Students

Country	Time Spent Working at a Paid Job per Week							
	No Time		Less than 5 Hours		5 to 10 Hours		More than 10 Hours	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
France	96 (0.3)	463 (3.1)	2 (0.2)	~ ~	1 (0.2)	~ ~	1 (0.1)	~ ~
Italy	91 (0.8)	423 (5.3)	3 (0.4)	440 (13.3)	4 (0.4)	414 (15.5)	3 (0.4)	388 (17.6)
Lebanon	92 (1.1)	536 (3.5)	2 (0.5)	~ ~	2 (0.5)	~ ~	3 (0.7)	508 (17.6)
Norway	47 (1.7)	467 (5.2)	13 (1.0)	467 (5.9)	23 (1.2)	457 (5.5)	18 (1.9)	439 (5.9)
Portugal	93 (0.5)	485 (2.5)	2 (0.3)	~ ~	3 (0.3)	467 (11.0)	3 (0.3)	443 (9.4)
Russian Federation	93 (0.4)	487 (5.7)	2 (0.2)	~ ~	2 (0.2)	~ ~	3 (0.2)	470 (9.8)
Russian Federation 6hr+	93 (0.6)	541 (7.7)	2 (0.4)	~ ~	2 (0.3)	~ ~	3 (0.4)	522 (17.2)
Slovenia	81 (0.8)	467 (3.2)	7 (0.5)	449 (8.0)	7 (0.5)	439 (8.3)	5 (0.5)	419 (8.2)
Sweden	71 (1.3)	430 (4.4)	11 (0.7)	450 (6.4)	13 (0.8)	443 (6.2)	6 (0.4)	411 (7.7)
United States	65 (1.8)	491 (6.4)	4 (0.5)	473 (15.4)	10 (1.0)	494 (9.1)	20 (1.2)	471 (7.0)
International Avg.	81 (0.4)	472 (1.5)	5 (0.2)	456 (4.7)	7 (0.2)	452 (4.0)	7 (0.3)	444 (4.0)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

**Exhibit M4.3: Students Speak the Language of the Test at Home**

Reported by Advanced Mathematics Students

Country	Always		Almost Always		Sometimes		Never	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
France	89 (0.9)	465 (2.9)	8 (0.7)	446 (7.4)	2 (0.3)	~ ~	1 (0.2)	~ ~
Italy	72 (1.4)	439 (4.9)	19 (1.2)	386 (8.2)	7 (0.7)	345 (15.1)	1 (0.3)	~ ~
Lebanon	5 (0.7)	527 (12.6)	12 (1.2)	540 (7.8)	65 (1.7)	529 (3.7)	19 (1.6)	538 (5.4)
Norway	85 (1.4)	462 (4.7)	9 (1.0)	449 (8.5)	4 (0.5)	433 (7.5)	2 (0.4)	~ ~
Portugal	91 (0.6)	483 (2.6)	7 (0.5)	478 (5.5)	2 (0.2)	~ ~	0 (0.1)	~ ~
Russian Federation	86 (1.2)	485 (5.7)	11 (0.8)	487 (9.1)	2 (0.6)	~ ~	0 (0.2)	~ ~
Russian Federation 6hr+	85 (1.2)	540 (8.4)	13 (1.0)	547 (8.5)	2 (0.3)	~ ~	0 (0.1)	~ ~
Slovenia	88 (0.7)	462 (3.6)	9 (0.7)	454 (8.0)	2 (0.3)	~ ~	1 (0.2)	~ ~
Sweden	78 (1.4)	444 (3.8)	12 (0.7)	401 (6.9)	8 (0.9)	368 (11.8)	2 (0.3)	~ ~
United States	74 (2.4)	491 (4.4)	16 (1.5)	484 (12.0)	8 (0.9)	463 (12.0)	3 (0.9)	423 (50.0)
International Avg.	74 (0.4)	473 (1.9)	12 (0.3)	458 (2.8)	11 (0.3)	428 (4.8)	3 (0.2)	481 (25.2)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

**Exhibit M4.4: Students' Expectations for Further Education**

Reported by Advanced Mathematics Students

Country	Doctoral Degree		Master's Degree		Bachelor's Degree		Post-Secondary Education but Not Bachelor's Degree		Upper-Secondary Education	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
Lebanon	58 (2.1)	538 (3.9)	35 (2.0)	530 (4.3)	4 (0.6)	515 (9.5)	2 (0.3)	~ ~	1 (0.4)	~ ~
Portugal	24 (0.9)	504 (3.2)	47 (0.9)	495 (3.0)	23 (0.8)	450 (4.1)	3 (0.3)	430 (8.2)	3 (0.4)	408 (8.2)
United States	23 (1.1)	478 (11.3)	49 (1.2)	494 (5.2)	27 (1.3)	477 (6.5)	0 (0.1)	~ ~	0 (0.1)	~ ~
France	21 (0.8)	476 (3.9)	54 (1.0)	475 (3.3)	13 (0.8)	433 (4.9)	11 (0.6)	421 (4.2)	2 (0.4)	~ ~
Slovenia	18 (1.5)	499 (6.3)	48 (1.0)	469 (3.2)	22 (1.1)	434 (5.0)	12 (0.9)	416 (6.7)	1 (0.2)	~ ~
Italy	14 (0.7)	464 (7.1)	19 (0.9)	434 (6.5)	41 (1.1)	442 (5.6)	15 (0.9)	378 (8.5)	11 (1.0)	337 (16.6)
Russian Federation 6hr+	12 (1.4)	586 (10.4)	62 (1.1)	545 (6.7)	25 (1.2)	512 (11.1)	1 (0.3)	~ ~	0 (0.1)	~ ~
Sweden	9 (0.7)	455 (8.4)	61 (1.1)	451 (4.1)	26 (1.1)	391 (4.9)	4 (0.5)	375 (9.8)	0 (0.1)	~ ~
Russian Federation	8 (0.6)	529 (9.9)	58 (1.1)	492 (5.4)	31 (1.0)	467 (7.3)	2 (0.3)	~ ~	1 (0.3)	~ ~
Norway	8 (0.8)	495 (5.6)	70 (1.7)	469 (4.8)	20 (1.5)	419 (4.7)	1 (0.4)	~ ~	1 (0.2)	~ ~
International Avg.	20 (0.4)	493 (2.4)	49 (0.4)	479 (1.5)	23 (0.4)	448 (2.0)	6 (0.2)	404 (3.4)	2 (0.1)	373 (9.2)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Students' Expectations for Further Education by Gender**

Reported by Advanced Mathematics Students

Country	Doctoral Degree		Master's Degree		Bachelor's Degree		Post-Secondary Education but Not Bachelor's Degree		Upper-Secondary Education	
	Percent of Females	Percent of Males	Percent of Females	Percent of Males	Percent of Females	Percent of Males	Percent of Females	Percent of Males	Percent of Females	Percent of Males
Lebanon	65 (3.7) ♀	54 (2.3)	32 (3.6)	37 (2.2)	3 (0.8)	5 (0.7) ♀	0 (0.2)	2 (0.5) ♀	0 (0.2)	2 (0.6) ♀
Portugal	27 (1.3) ♀	21 (1.3)	49 (1.3) ♀	45 (1.2)	21 (1.1)	27 (1.2) ♀	1 (0.2)	4 (0.7) ♀	2 (0.4)	3 (0.6)
United States	25 (2.1)	21 (1.6)	51 (2.1)	48 (2.9)	24 (1.6)	30 (2.2) ♀	0 (0.1)	0 (0.2)	0 (0.1)	0 (0.1)
France	26 (1.2) ♀	16 (1.0)	47 (1.3)	60 (1.2) ♀	13 (1.0)	13 (1.0)	12 (1.0) ♀	9 (0.8)	1 (0.4)	2 (0.4)
Slovenia	17 (1.6)	19 (1.7)	53 (1.5) ♀	40 (1.4)	21 (1.4)	24 (1.4)	9 (1.2)	16 (1.3) ♀	0 (0.3)	1 (0.3)
Italy	13 (1.2)	14 (0.9)	26 (1.4) ♀	15 (1.0)	44 (1.6) ♀	38 (1.5)	14 (1.4)	16 (1.2)	3 (0.4)	17 (1.6) ♀
Russian Federation 6hr+	10 (1.0)	15 (2.0) ♀	64 (1.5)	60 (1.9)	26 (1.7)	24 (1.3)	1 (0.3)	1 (0.3)	0 (0.1)	0 (0.1)
Sweden	10 (1.1) ♀	7 (0.6)	63 (1.6)	59 (1.4)	25 (1.5)	27 (1.3)	2 (0.4)	6 (0.6) ♀	0 (0.1)	1 (0.2) ♀
Russian Federation	7 (0.7)	9 (0.8) ♀	59 (1.4)	57 (1.2)	31 (1.2)	30 (1.2)	2 (0.7)	2 (0.4)	1 (0.3)	1 (0.4)
Norway	8 (0.8)	8 (1.1)	75 (1.6) ♀	67 (2.1)	17 (1.5)	22 (2.0) ♀	0 (0.2)	2 (0.5) ♀	0 (0.2)	1 (0.3) ♀
International Avg.	22 (0.6) ♀	19 (0.5)	50 (0.6) ♀	47 (0.6)	22 (0.4)	24 (0.5) ♀	5 (0.3)	6 (0.3) ♀	1 (0.1)	3 (0.2) ♀

♀ Percent significantly higher than other gender

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M4.5: Intended Areas of Study for Students Planning to Continue Their Education**

Reported by Advanced Mathematics Students

Students could indicate more than one area of study.

Country	Mathematics or Statistics		Physics		Engineering and Engineering Technologies		Computer and Information Sciences		Chemistry	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
France	15 (0.7)	520 (4.5)	14 (0.6)	512 (4.5)	23 (0.8)	499 (4.3)	12 (0.6)	490 (4.9)	11 (0.6)	485 (4.6)
Italy	9 (0.6)	473 (12.2)	6 (0.5)	514 (12.2)	25 (1.2)	467 (8.3)	7 (0.8)	429 (11.5)	8 (0.6)	445 (10.4)
Lebanon	23 (1.4)	529 (5.0)	18 (1.3)	531 (7.0)	75 (1.5)	539 (3.9)	12 (0.9)	530 (7.0)	5 (0.8)	525 (15.0)
Norway	31 (1.1)	491 (4.5)	33 (1.2)	494 (4.8)	68 (1.4)	466 (5.3)	23 (1.0)	466 (6.4)	15 (1.2)	477 (6.4)
Portugal	10 (0.6)	526 (4.2)	7 (0.6)	541 (6.2)	28 (1.3)	508 (3.7)	12 (0.8)	490 (7.0)	5 (0.5)	524 (6.6)
Russian Federation	28 (1.2)	529 (6.0)	28 (1.4)	520 (6.8)	27 (1.3)	511 (6.8)	24 (1.2)	519 (6.3)	9 (0.6)	498 (7.5)
Russian Federation 6hr+	41 (1.9)	575 (7.1)	36 (1.4)	572 (7.3)	34 (1.4)	562 (7.2)	32 (1.4)	573 (6.7)	7 (1.0)	539 (14.0)
Slovenia	6 (0.5)	528 (8.6)	5 (0.5)	523 (7.4)	14 (1.0)	485 (4.3)	8 (0.6)	486 (7.6)	7 (0.7)	500 (6.6)
Sweden	18 (0.9)	487 (4.9)	15 (0.9)	492 (5.9)	47 (0.9)	453 (4.3)	18 (1.1)	429 (5.8)	9 (0.5)	451 (8.0)
United States	26 (1.1)	512 (10.8)	15 (1.0)	525 (16.7)	28 (1.6)	523 (5.9)	17 (1.4)	502 (14.8)	14 (1.0)	524 (9.3)
International Avg.	18 (0.3)	511 (2.4)	16 (0.3)	517 (2.9)	37 (0.4)	494 (1.8)	15 (0.3)	482 (2.8)	9 (0.3)	492 (2.9)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

Country	Biological and Biomedical Sciences		Education		Business		Other	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
France	40 (1.0)	449 (3.1)	7 (0.5)	452 (5.9)	14 (0.6)	458 (5.1)	30 (0.9)	448 (3.4)
Italy	33 (1.2)	440 (6.0)	12 (0.8)	414 (9.7)	13 (0.8)	444 (7.2)	38 (1.0)	406 (6.1)
Lebanon	5 (0.7)	535 (12.7)	2 (0.4)	~ ~	6 (0.9)	530 (14.0)	15 (1.1)	520 (5.5)
Norway	24 (0.9)	453 (5.3)	11 (0.7)	457 (5.7)	25 (1.8)	450 (5.1)	34 (1.1)	453 (4.4)
Portugal	38 (1.7)	494 (2.7)	3 (0.2)	441 (7.7)	26 (1.7)	467 (4.4)	30 (1.1)	461 (3.0)
Russian Federation	15 (0.8)	475 (8.4)	11 (0.6)	472 (9.4)	29 (0.9)	483 (6.8)	50 (1.0)	475 (5.8)
Russian Federation 6hr+	9 (0.9)	512 (19.2)	8 (0.6)	532 (10.6)	32 (1.5)	532 (7.8)	47 (1.4)	527 (8.0)
Slovenia	23 (1.2)	497 (5.7)	12 (0.8)	435 (4.4)	10 (0.8)	424 (7.1)	41 (1.3)	433 (3.8)
Sweden	32 (1.2)	424 (6.4)	6 (0.5)	433 (8.0)	16 (0.8)	416 (5.8)	32 (1.0)	411 (4.9)
United States	33 (1.2)	479 (6.1)	8 (0.6)	460 (7.9)	25 (1.4)	485 (6.0)	42 (1.5)	477 (6.0)
International Avg.	27 (0.4)	472 (2.3)	8 (0.2)	445 (2.7)	18 (0.4)	462 (2.5)	35 (0.4)	454 (1.6)

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.



**Exhibit M4.6: Students' Intended Future Profession**

Reported by Advanced Mathematics Students

Students indicated either "yes" or "maybe" when asked if they wanted to work in the professional fields shown below.

Country	Engineering and Engineering Technologies		Computer and Information Sciences		Biological and Biomedical Sciences		Environmental Sciences	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
France	48 (1.0)	487 (3.4)	28 (0.9)	488 (3.7)	59 (1.0)	455 (2.9)	35 (0.9)	465 (3.5)
Italy	43 (1.4)	444 (7.1)	32 (1.5)	424 (7.9)	48 (1.2)	437 (5.6)	33 (1.2)	424 (6.8)
Lebanon	93 (0.8)	535 (3.1)	61 (2.1)	536 (4.2)	23 (1.9)	534 (7.0)	24 (1.6)	535 (6.2)
Norway	89 (0.7)	462 (4.9)	55 (1.5)	465 (5.6)	53 (1.3)	457 (4.7)	59 (1.4)	467 (4.6)
Portugal	43 (1.4)	497 (3.9)	37 (1.1)	485 (4.0)	49 (1.6)	491 (2.8)	28 (1.1)	474 (3.2)
Russian Federation	55 (1.5)	504 (5.9)	55 (1.4)	502 (6.0)	32 (0.9)	476 (6.7)	27 (1.0)	477 (6.5)
Russian Federation 6hr+	62 (1.6)	557 (7.2)	62 (1.5)	555 (7.3)	25 (1.5)	529 (11.6)	23 (1.1)	527 (10.7)
Slovenia	39 (1.2)	486 (4.5)	34 (1.3)	475 (4.9)	53 (1.2)	483 (5.2)	44 (1.2)	464 (4.2)
Sweden	79 (0.8)	442 (4.2)	55 (1.4)	438 (4.6)	61 (1.1)	430 (4.7)	49 (1.3)	438 (4.0)
United States	52 (1.4)	508 (7.2)	40 (1.6)	500 (8.3)	54 (1.1)	481 (6.6)	30 (1.9)	486 (9.0)
International Avg.	60 (0.4)	485 (1.7)	44 (0.5)	479 (1.9)	48 (0.4)	472 (1.8)	37 (0.4)	470 (1.9)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

Country	Agriculture and Agricultural Sciences		Education		Finance/Banking		Actuarial Sciences	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
France	11 (0.6)	464 (5.2)	46 (0.9)	471 (3.3)	28 (0.9)	466 (4.7)	16 (0.6)	461 (4.2)
Italy	21 (0.9)	414 (9.3)	41 (1.3)	435 (5.5)	41 (1.1)	417 (7.0)	20 (1.0)	410 (7.7)
Lebanon	19 (1.7)	523 (6.2)	57 (2.3)	540 (4.4)	31 (2.3)	524 (7.7)	19 (2.0)	523 (7.2)
Norway	17 (1.1)	453 (4.7)	56 (1.5)	467 (4.6)	45 (2.3)	456 (4.0)	32 (1.3)	457 (5.8)
Portugal	16 (0.7)	468 (4.0)	27 (0.9)	494 (3.9)	37 (1.6)	480 (3.4)	9 (0.5)	476 (5.4)
Russian Federation	20 (0.9)	472 (7.3)	40 (0.9)	493 (6.9)	64 (1.1)	480 (6.4)	27 (0.8)	493 (6.2)
Russian Federation 6hr+	16 (1.1)	522 (11.8)	38 (1.4)	550 (9.4)	65 (1.8)	540 (7.9)	28 (1.4)	544 (10.1)
Slovenia	23 (1.1)	468 (4.7)	59 (1.2)	465 (3.9)	40 (1.2)	457 (4.1)	18 (1.0)	458 (6.5)
Sweden	18 (0.7)	425 (6.1)	46 (1.0)	448 (3.8)	48 (1.2)	426 (4.6)	24 (0.8)	448 (6.1)
United States	16 (1.3)	467 (14.8)	46 (1.4)	492 (5.2)	39 (1.6)	479 (8.8)	25 (1.3)	508 (11.1)
International Avg.	18 (0.3)	462 (2.5)	46 (0.4)	478 (1.6)	42 (0.5)	465 (2.0)	21 (0.4)	470 (2.3)

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

An "r" indicates data are available for at least 70% but less than 85% of the students. An "s" indicates data are available for at least 50% but less than 70% of the students.

**Exhibit M4.7: Students' Intended Future Profession by Gender**

Reported by Advanced Mathematics Students

Students indicated either "yes" or "maybe" when asked if they wanted to work in the professional fields shown below. The Percent of Females column shows the percent of female advanced mathematics students choosing that professional field and the Percent of Males column shows the percent of male advanced mathematics students choosing that professional field.

Country	Engineering and Engineering Technologies		Computer and Information Sciences		Biological and Biomedical Sciences		Environmental Sciences	
	Percent of Females	Percent of Males	Percent of Females	Percent of Males	Percent of Females	Percent of Males	Percent of Females	Percent of Males
France	32 (1.2)	63 (1.2) ▲	10 (0.7)	45 (1.3) ▲	72 (1.2) ▲	46 (1.3)	35 (1.2)	35 (1.2)
Italy	25 (1.7)	54 (1.6) ▲	13 (1.1)	43 (1.9) ▲	62 (1.6) ▲	40 (1.4)	31 (2.0)	34 (1.4)
Lebanon	89 (1.7)	95 (0.9) ▲ r	54 (3.7) s	66 (2.4) ▲ s	25 (3.0) s	22 (2.1) s	24 (2.8) s	23 (1.8) s
Norway	83 (1.3)	92 (0.6) ▲	35 (1.9)	68 (1.8) ▲	71 (1.6) ▲	41 (2.0)	67 (1.9) ▲	54 (1.9)
Portugal	25 (1.3)	60 (2.0) ▲	17 (1.0)	58 (1.5) ▲	64 (2.0) ▲	33 (1.8)	34 (1.4) ▲	22 (1.5)
Russian Federation	37 (1.8)	73 (1.6) ▲	39 (1.3)	71 (1.6) ▲	40 (1.1) ▲	24 (1.1)	34 (1.4) ▲	21 (1.0)
Russian Federation 6hr+	46 (2.4)	76 (1.8) ▲	48 (2.0)	75 (1.5) ▲	29 (1.9) ▲	22 (1.5)	30 (2.0) ▲	18 (1.3)
Slovenia	24 (1.0)	61 (2.0) ▲	20 (1.4)	55 (1.7) ▲	53 (1.5)	54 (1.6)	47 (1.6) ▲	41 (1.6)
Sweden	68 (1.5)	87 (0.8) ▲	32 (1.8)	70 (1.4) ▲	78 (1.2) ▲	50 (1.5)	56 (1.9) ▲	44 (1.8)
United States	35 (2.2)	68 (1.4) ▲	25 (2.0)	54 (2.1) ▲	60 (1.9) ▲	47 (1.9)	31 (2.7)	29 (2.2)
International Avg.	46 (0.5)	73 (0.5) ▲	27 (0.6)	59 (0.6) ▲	58 (0.6) ▲	40 (0.6)	40 (0.6) ▲	34 (0.5)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

Country	Agriculture and Agricultural Sciences		Education		Finance/Banking		Actuarial Sciences	
	Percent of Females	Percent of Males	Percent of Females	Percent of Males	Percent of Females	Percent of Males	Percent of Females	Percent of Males
France	12 (0.7)	10 (0.8)	49 (1.3) ▲	43 (1.3)	21 (1.1)	34 (1.2) ▲	15 (0.9)	18 (0.8) ▲
Italy	13 (1.2)	25 (1.3) ▲	46 (2.2) ▲	38 (1.5)	33 (1.7)	46 (1.3) ▲	17 (1.3)	21 (1.2) ▲
Lebanon	s 18 (2.2) s	s 20 (2.0) r	r 64 (2.4) ▲ r	s 52 (3.1) s	s 30 (4.0) s	s 32 (2.9) s	s 17 (2.9) s	s 21 (2.4) s
Norway	19 (1.6) ▲	15 (1.2)	57 (1.9)	55 (1.7)	37 (2.3)	50 (2.7) ▲	31 (2.0)	33 (1.4)
Portugal	14 (0.9)	18 (1.0) ▲	28 (1.2)	25 (1.4)	37 (2.1)	38 (1.9)	9 (0.8)	8 (0.8)
Russian Federation	18 (1.2)	22 (1.0) ▲	51 (1.2) ▲	29 (1.2)	64 (1.2)	64 (1.7)	27 (0.9)	26 (1.0)
Russian Federation 6hr+	14 (1.6)	18 (1.2) ▲	48 (2.2) ▲	29 (1.7)	65 (2.0)	66 (2.1)	30 (1.7)	26 (1.6)
Slovenia	21 (1.4)	26 (1.4) ▲	63 (1.4) ▲	53 (2.1)	35 (1.4)	46 (1.7) ▲	17 (1.5)	18 (1.3)
Sweden	19 (1.2)	17 (0.9)	49 (1.5) ▲	44 (1.5)	39 (1.7)	54 (1.4) ▲	19 (1.3)	27 (1.2) ▲
United States	14 (1.4)	18 (1.9)	48 (2.1)	43 (2.0)	33 (1.5)	45 (2.8) ▲	20 (1.6)	30 (2.0) ▲
International Avg.	17 (0.5)	19 (0.4) ▲	50 (0.6) ▲	42 (0.6)	37 (0.7)	46 (0.7) ▲	19 (0.5)	22 (0.5) ▲

▲ Percent significantly higher than other gender

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

An "r" indicates data are available for at least 70% but less than 85% of the students. An "s" indicates data are available for at least 50% but less than 70% of the students.





# CHAPTER M5: SCHOOL COMPOSITION

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS

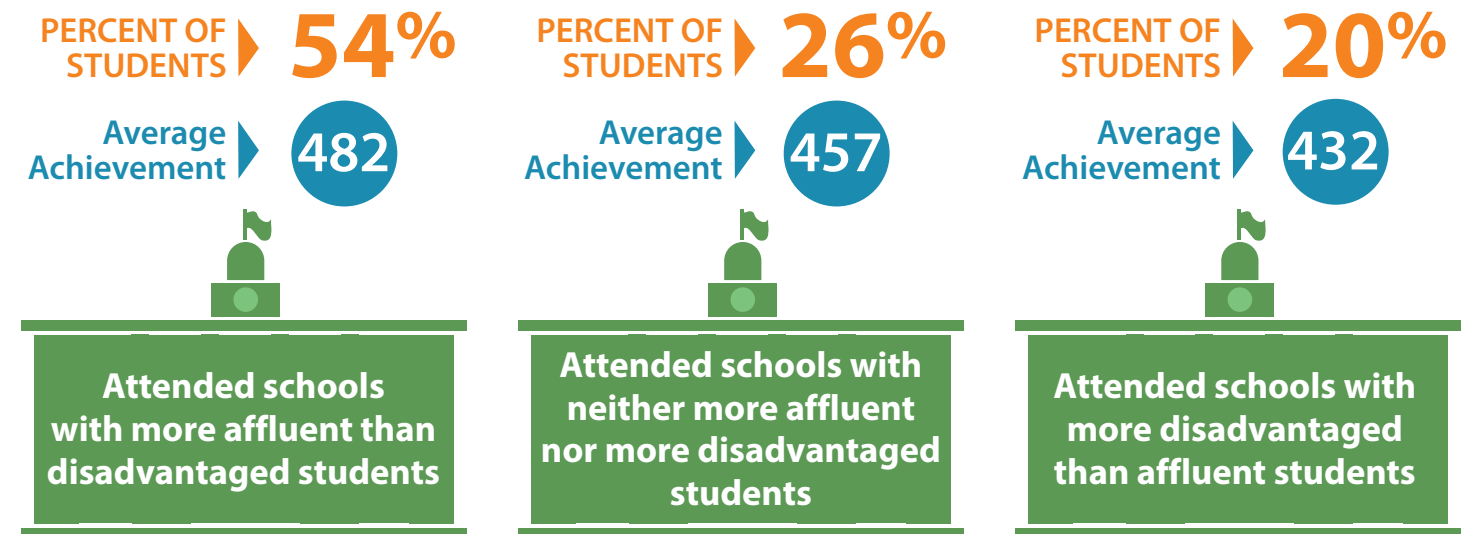


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Lynch School of Education, Boston College



## Socioeconomic Composition of Schools



**In nearly all the TIMSS Advanced countries, students attending schools with more affluent than disadvantaged students had higher average mathematics achievement.**

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015.

<http://timss2015.org/advanced/download-center/>



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**Exhibit M5.1: School Composition by Economic Background of the Student Body**

Reported by Principals

Country	More Affluent - Schools where more than 25% of the student body comes from economically affluent homes and not more than 25% from economically disadvantaged homes		Neither More Affluent nor More Disadvantaged		More Disadvantaged - Schools where more than 25% of the student body comes from economically disadvantaged homes and not more than 25% from economically affluent homes	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
France	50 (4.1)	471 (4.0)	28 (3.9)	462 (5.5)	22 (3.4)	437 (6.1)
Italy	48 (4.2)	442 (7.4)	40 (4.8)	423 (11.0)	12 (3.4)	331 (17.7)
Lebanon	34 (5.1)	554 (7.1)	29 (4.6)	529 (8.4)	37 (3.1)	515 (4.0)
Norway <sub>r</sub>	77 (5.5)	468 (6.1)	22 (5.5)	447 (6.5)	1 (0.9)	~ ~
Portugal <sub>r</sub>	18 (3.3)	490 (6.5)	36 (3.5)	479 (4.5)	46 (4.1)	476 (4.6)
Russian Federation	80 (3.2)	492 (6.1)	15 (2.5)	454 (15.6)	5 (1.4)	442 (40.2)
Russian Federation 6hr+	91 (2.4)	546 (8.2)	8 (2.3)	507 (29.7)	1 (0.7)	~ ~
Slovenia	64 (5.0)	474 (5.0)	24 (4.9)	438 (8.1)	12 (2.2)	427 (11.9)
Sweden <sub>r</sub>	80 (3.7)	442 (4.1)	13 (2.6)	379 (12.9)	7 (2.7)	374 (13.6)
United States	34 (4.6)	509 (10.8)	28 (4.2)	498 (8.6)	38 (3.9)	458 (9.4)
International Avg.	54 (1.4)	482 (2.2)	26 (1.4)	457 (3.2)	20 (1.0)	432 (6.1)

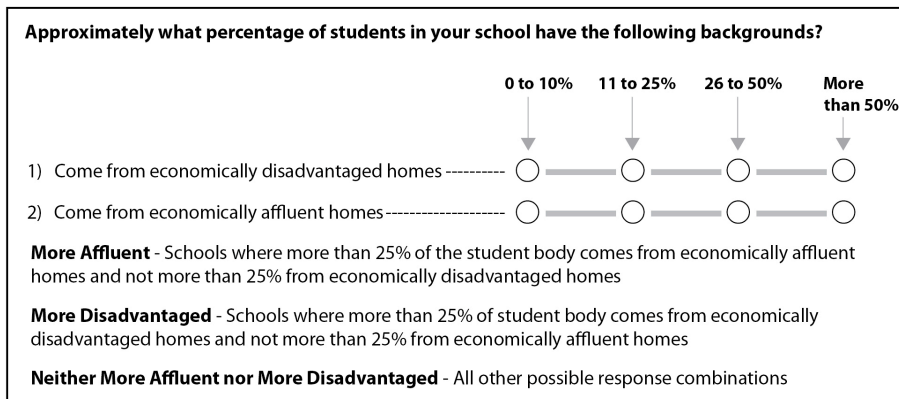
SOURCE: IEA's Trends in International Mathematics and Science Study - TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

An "r" indicates data are available for at least 70% but less than 85% of the students.





**Exhibit M5.2: Schools with Students Having the Language of the Test as Their Native Language**

Reported by Principals

Country	School Has More than 90% of Students with Language of Test as Their Native Language		School Has 51-90% of Students with Language of Test as Their Native Language		School Has 50% or Less of Students with Language of Test as Their Native Language	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
France	88 (3.1)	462 (3.1)	11 (3.1)	451 (14.3)	1 (0.6)	~ ~
Italy	83 (3.2)	426 (5.6)	17 (3.1)	402 (17.7)	1 (0.7)	~ ~
Lebanon	11 (3.6)	533 (9.0)	12 (2.7)	511 (7.7)	76 (4.3)	534 (4.3)
Norway <sup>r</sup>	72 (8.0)	466 (6.2)	28 (8.0)	452 (11.4)	0 (0.0)	~ ~
Portugal	92 (2.0)	482 (2.6)	5 (1.2)	468 (9.3)	4 (1.7)	504 (11.2)
Russian Federation	82 (2.8)	485 (6.0)	12 (2.2)	482 (19.4)	5 (1.3)	488 (31.7)
Russian Federation 6hr+	91 (3.5)	541 (8.2)	6 (3.2)	516 (71.0)	3 (1.5)	549 (12.7)
Slovenia	88 (3.8)	462 (3.8)	12 (3.8)	443 (12.5)	0 (0.0)	~ ~
Sweden	31 (5.2)	450 (5.9)	58 (6.2)	424 (6.0)	11 (4.0)	414 (19.7)
United States	54 (4.3)	502 (5.5)	32 (3.7)	485 (8.4)	14 (3.8)	425 (18.1)
International Avg.	67 (1.4)	474 (1.9)	21 (1.4)	458 (4.2)	13 (0.8)	473 (8.6)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

An "r" indicates data are available for at least 70% but less than 85% of the students.



# CHAPTER M6: SCHOOL CLIMATE

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS



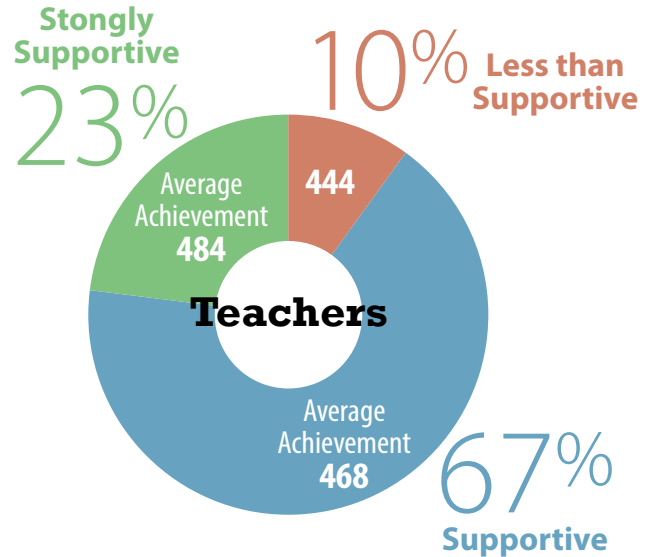
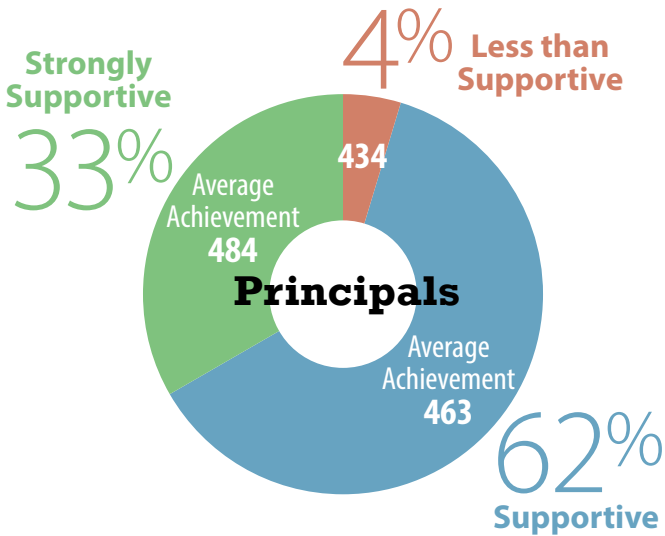
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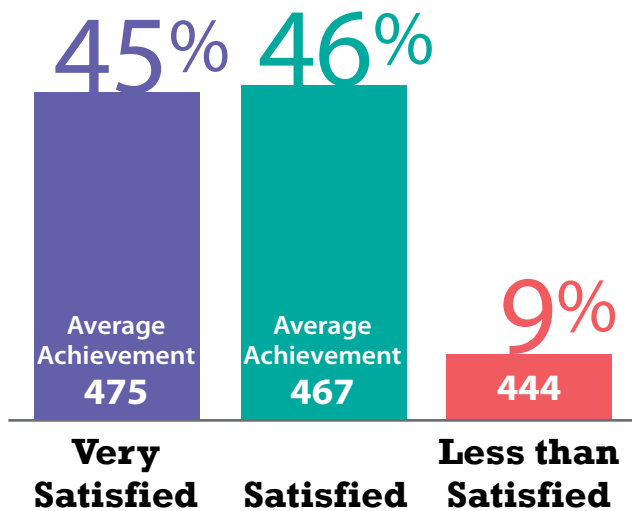
## Schools Have Positive Environments

Generally, students taking advanced mathematics courses are in positive school environments, and the more positive the school environment the higher the average achievement.

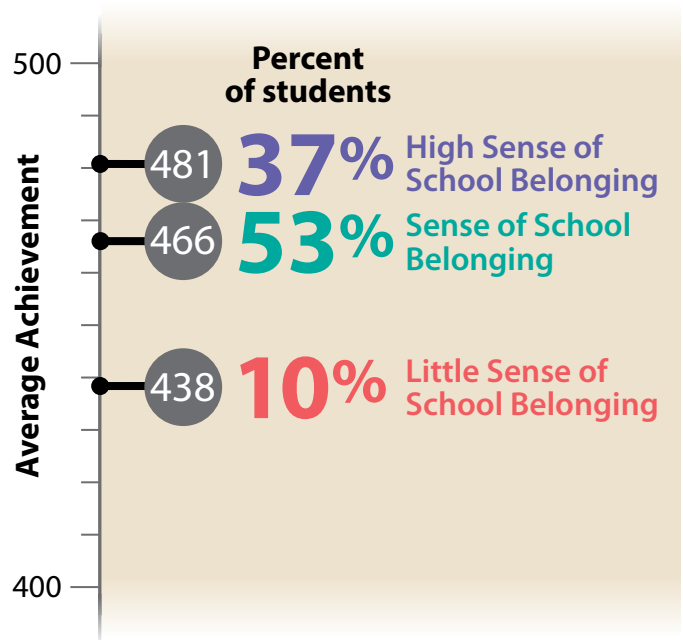


**Principals and teachers agree that high percentages of TIMSS Advanced students attend schools that support advanced mathematics education, although the principals have more positive attitudes than the teachers.**

**TEACHERS** of advanced mathematics reported a high degree of job satisfaction. Almost all students (91%) had teachers who were very satisfied or satisfied with their careers.



**STUDENTS** of advanced mathematics reported a positive sense of school belonging.





**Exhibit M6.1: Programs to Encourage Students to Study Advanced Mathematics**

*Reported by National Research Coordinators*

Country	School Partnerships with Industry	School Collaborations with Universities	Contests/Competitions in Advanced Mathematics
France	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
Italy	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
Lebanon	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Norway	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
Portugal	<input type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
Russian Federation	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
Slovenia	<input type="radio"/>	<input type="radio"/>	<input checked="" type="radio"/>
Sweden	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>
United States	<input checked="" type="radio"/>	<input checked="" type="radio"/>	<input checked="" type="radio"/>

● Yes  
○ No

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M6.2: School Supports Advanced Mathematics and Physics Education – Principal Version**

Reported by Principals

Students were scored according to their principals' degree of agreement with seven statements on the *School Supports Advanced Mathematics and Physics Education* scale. Students in schools where their principals reported that the school is **Strongly Supportive** of advanced mathematics and physics education had a score on the scale of at least 11.0, which corresponds to their principals "agreeing a lot" with four of the seven statements and "agreeing a little" with the other three, on average. Students in schools that are **Less than Supportive** of advanced mathematics and physics education had a score no higher than 6.5, which corresponds to their principals "disagreeing a little" with four of the seven statements and "agreeing a little" with the other three, on average. All other students attended schools that are **Supportive** of advanced mathematics and physics education.

Country	Strongly Supportive		Supportive		Less than Supportive		Average Scale Score
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	
Russian Federation 6hr+	86 (2.4)	544 (8.6)	14 (2.4)	513 (18.1)	0 (0.0)	~ ~	12.3 (0.13)
Russian Federation	73 (3.2)	494 (7.5)	27 (3.2)	460 (11.2)	0 (0.0)	~ ~	11.8 (0.13)
Norway	62 (7.2)	474 (6.7)	38 (7.2)	442 (5.3)	0 (0.3)	~ ~	11.3 (0.24)
United States	47 (5.2)	495 (7.9)	51 (5.2)	481 (8.5)	3 (1.1)	435 (20.9)	10.6 (0.15)
Lebanon	39 (2.5)	536 (4.8)	60 (2.6)	531 (4.0)	2 (0.3)	~ ~	10.6 (0.10)
Portugal	35 (3.7)	483 (3.4)	61 (4.0)	484 (3.3)	4 (1.6)	470 (11.0)	10.2 (0.12)
Italy	21 (4.2)	437 (13.6)	72 (4.6)	417 (8.1)	6 (2.5)	413 (26.9)	9.5 (0.17)
France	8 (2.3)	479 (11.0)	84 (3.4)	460 (3.2)	8 (2.5)	456 (10.0)	8.7 (0.14)
Slovenia	7 (4.4)	505 (44.9)	89 (5.2)	458 (5.2)	4 (2.8)	410 (46.0)	8.9 (0.14)
Sweden	6 (3.0)	456 (13.6)	80 (4.3)	431 (4.6)	13 (3.1)	419 (13.8)	8.6 (0.14)
International Avg.	33 (1.4)	484 (5.8)	62 (1.5)	463 (2.2)	4 (0.6)	434 (10.1)	

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

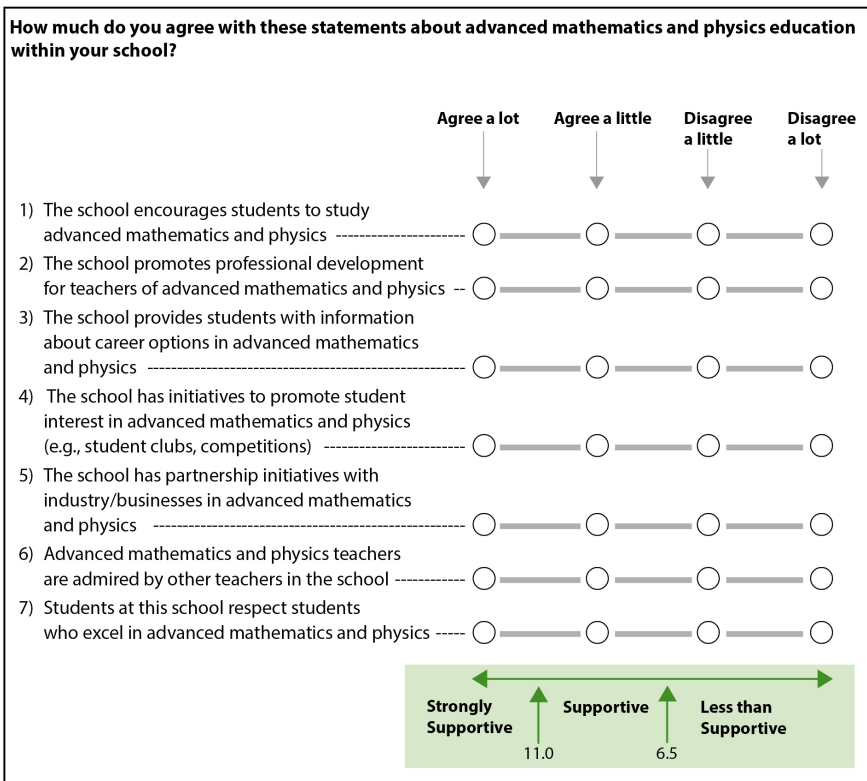
The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

An "r" indicates data are available for at least 70% but less than 85% of the students.



**Exhibit M6.3: School Supports Advanced Mathematics and Physics Education – Teacher Version**

Reported by Advanced Mathematics Teachers

Students were scored according to their teachers' degree of agreement with seven statements on the *School Supports Advanced Mathematics and Physics Education* scale. Students in schools where their teachers reported that the school is **Strongly Supportive** of advanced mathematics and physics education had a score on the scale of at least 11.6, which corresponds to their teachers "agreeing a lot" with four of the seven statements and "agreeing a little" with the other three, on average. Students in schools that are **Less than Supportive** of advanced mathematics and physics education had a score no higher than 7.4, which corresponds to their teachers "disagreeing a little" with four of the seven statements and "agreeing a little" with the other three, on average. All other students attended schools that are **Supportive** of advanced mathematics and physics education.

Country	Strongly Supportive		Supportive		Less than Supportive		Average Scale Score
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	
Russian Federation 6hr+	62 (4.4)	549 (6.9)	38 (4.4)	525 (18.4)	0 (0.0)	~ ~	12.1 (0.15)
Russian Federation	52 (2.5)	503 (8.6)	48 (2.5)	464 (7.9)	1 (0.5)	~ ~	11.5 (0.11)
Lebanon	46 (3.2)	533 (4.1)	52 (3.3)	532 (5.0)	1 (0.2)	~ ~	11.5 (0.10)
United States <sup>r</sup>	40 (4.0)	487 (11.4)	56 (3.9)	487 (5.5)	4 (1.1)	433 (25.3)	10.9 (0.17)
Norway	28 (6.5)	476 (6.4)	70 (7.1)	456 (5.8)	3 (1.5)	463 (31.0)	10.6 (0.17)
Portugal	24 (3.1)	486 (5.0)	72 (3.5)	482 (3.0)	5 (1.5)	475 (10.1)	10.3 (0.12)
Italy	6 (1.7)	465 (30.4)	69 (3.3)	427 (6.6)	25 (3.2)	406 (14.5)	8.8 (0.13)
France	4 (1.2)	479 (12.0)	87 (2.1)	461 (3.2)	9 (1.7)	459 (6.0)	9.2 (0.08)
Sweden	3 (1.1)	442 (10.8)	77 (3.2)	435 (4.5)	20 (3.3)	436 (8.8)	8.8 (0.11)
Slovenia	2 (0.1)	~ ~	77 (2.7)	466 (4.6)	22 (2.7)	434 (7.4)	8.4 (0.08)
International Avg.	23 (1.1)	484 (4.8)	67 (1.2)	468 (1.8)	10 (0.7)	444 (6.5)	

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

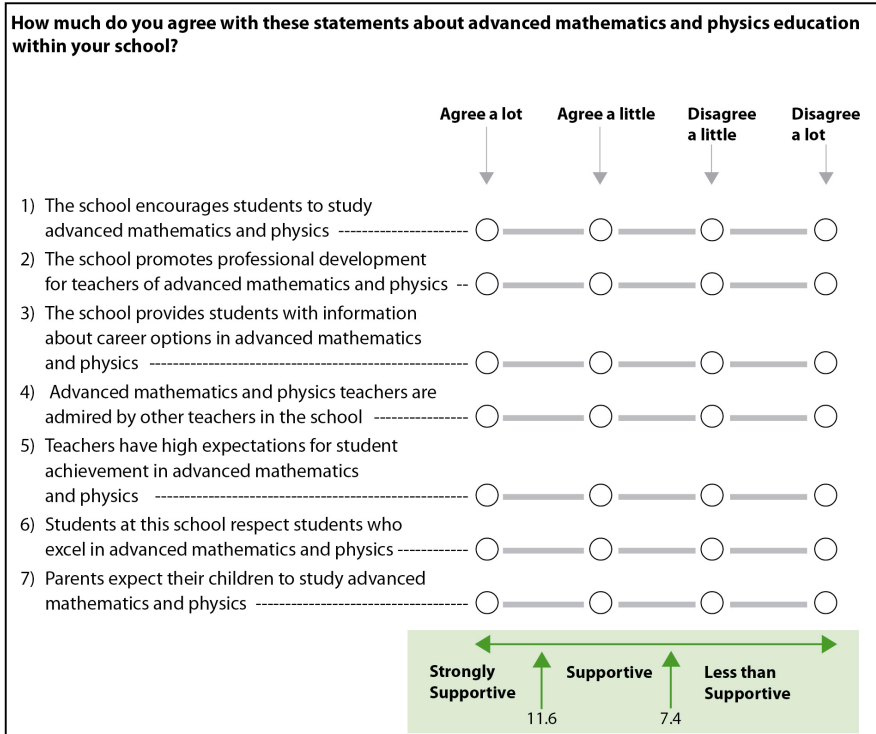
The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

An "r" indicates data are available for at least 70% but less than 85% of the students.





**Exhibit M6.4: Teacher Job Satisfaction**

Reported by Advanced Mathematics Teachers

Students were scored according to how often their teachers responded positively to the seven statements on the *Teacher Job Satisfaction* scale. Students with **Very Satisfied** teachers had a score on the scale of at least 10.5, which corresponds to their teachers responding "very often" to four of the seven statements and responding "often" to the other three, on average. Students with **Less than Satisfied** teachers had a score no higher than 7.2, which corresponds to their teachers responding "sometimes" to four of the seven statements and responding "often" to the other three, on average. All other students had **Satisfied** teachers.

Country	Very Satisfied		Satisfied		Less than Satisfied		Average Scale Score
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	
Lebanon	79 (3.5)	534 (3.7)	19 (3.5)	529 (6.8)	2 (0.9)	~ ~	11.4 (0.08)
United States <sup>r</sup>	63 (3.8)	481 (8.0)	32 (3.5)	489 (7.3)	5 (1.3)	493 (14.1)	10.8 (0.15)
Norway	60 (4.8)	473 (6.0)	36 (4.7)	446 (5.6)	4 (1.8)	423 (11.1)	10.6 (0.18)
Russian Federation 6hr+	54 (4.8)	540 (11.7)	44 (4.6)	543 (11.3)	3 (1.2)	556 (59.5)	10.5 (0.13)
Russian Federation	42 (2.9)	505 (7.3)	55 (3.0)	471 (8.4)	3 (1.0)	415 (26.7)	10.1 (0.11)
France	36 (2.9)	465 (4.5)	50 (2.8)	459 (4.3)	14 (2.3)	459 (7.3)	9.6 (0.14)
Portugal	34 (3.3)	486 (4.6)	55 (3.5)	481 (3.8)	12 (2.2)	477 (6.0)	9.6 (0.13)
Slovenia	31 (4.4)	476 (6.4)	56 (4.9)	452 (7.3)	13 (3.2)	453 (11.3)	9.5 (0.18)
Sweden	29 (3.5)	436 (6.4)	60 (4.2)	437 (5.8)	11 (2.3)	423 (9.0)	9.4 (0.14)
Italy	28 (3.1)	417 (11.2)	52 (3.4)	435 (9.1)	20 (2.9)	408 (11.1)	9.1 (0.16)
International Avg.	45 (1.2)	475 (2.3)	46 (1.3)	467 (2.2)	9 (0.7)	444 (4.8)	

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

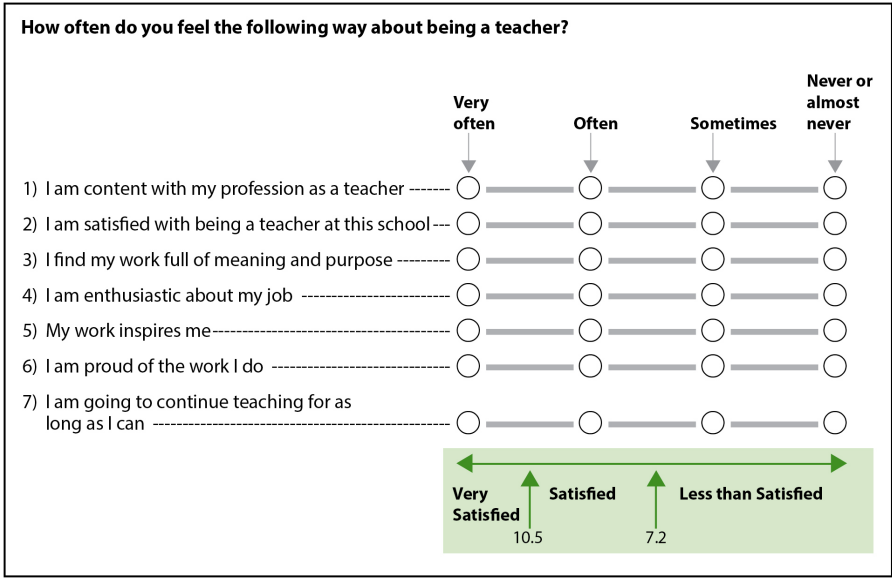
The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

An "r" indicates data are available for at least 70% but less than 85% of the students.



**Exhibit M6.5: Students' Sense of School Belonging**

Reported by Advanced Mathematics Students

Students were scored according to their agreement to nine statements about their *Sense of School Belonging*. Students with a **High Sense of School Belonging** had a score on the scale of at least 10.6, which corresponds to their "agreeing a lot" to five of the nine statements and "agreeing a little" to each of the other four statements, on average. Students with **Little Sense of School Belonging** had a score no higher than 7.7, which corresponds to their "disagreeing a little" to five of the nine statements and "agreeing a little" to each of the other four statements, on average. All other students had a **Sense of School Belonging**.

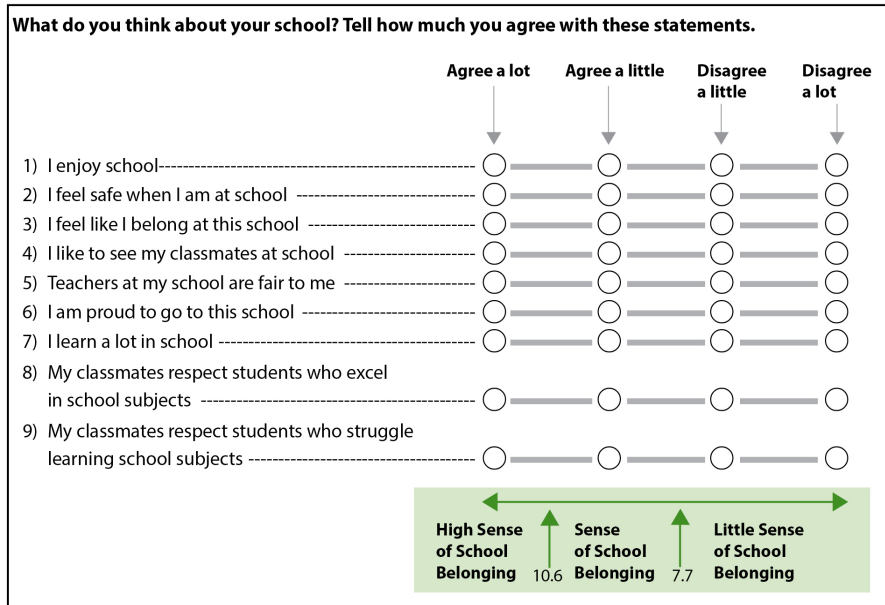
Country	High Sense of School Belonging		Sense of School Belonging		Little Sense of School Belonging		Average Scale Score
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	
Norway	59 (1.7)	468 (4.6)	38 (1.6)	450 (4.9)	3 (0.4)	420 (13.3)	11.0 (0.08)
Sweden	54 (1.4)	446 (4.1)	41 (1.3)	421 (5.0)	5 (0.6)	366 (9.2)	10.7 (0.06)
Russian Federation	49 (1.6)	494 (6.4)	43 (1.3)	480 (5.9)	8 (0.5)	450 (10.5)	10.6 (0.08)
Russian Federation 6hr+	49 (2.1)	547 (8.5)	43 (1.8)	537 (7.5)	8 (0.8)	517 (15.3)	10.6 (0.11)
United States	46 (2.0)	492 (6.9)	47 (1.8)	485 (5.3)	7 (0.7)	452 (7.6)	10.4 (0.09)
Lebanon	43 (2.1)	542 (4.1)	48 (1.9)	528 (4.3)	9 (1.0)	526 (8.5)	10.2 (0.08)
Portugal	38 (1.3)	489 (3.9)	55 (1.1)	480 (2.8)	7 (0.5)	466 (5.3)	10.1 (0.05)
France	20 (1.2)	479 (4.4)	73 (1.0)	463 (3.0)	7 (0.7)	421 (7.2)	9.4 (0.05)
Italy	16 (1.1)	424 (11.9)	61 (1.2)	427 (5.4)	23 (1.2)	408 (8.7)	8.9 (0.05)
Slovenia	11 (0.9)	493 (6.4)	69 (0.9)	464 (4.0)	20 (0.9)	430 (5.0)	8.8 (0.05)
International Avg.	37 (0.5)	481 (2.1)	53 (0.5)	466 (1.5)	10 (0.3)	438 (2.9)	

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.







# CHAPTER M7: SCHOOL SAFETY

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS



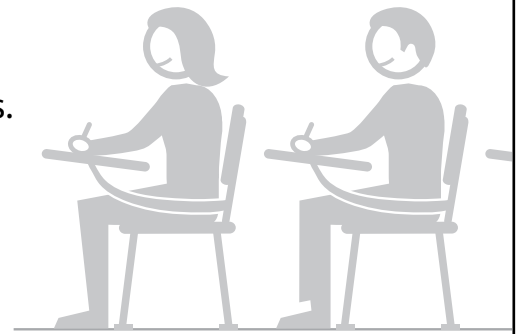
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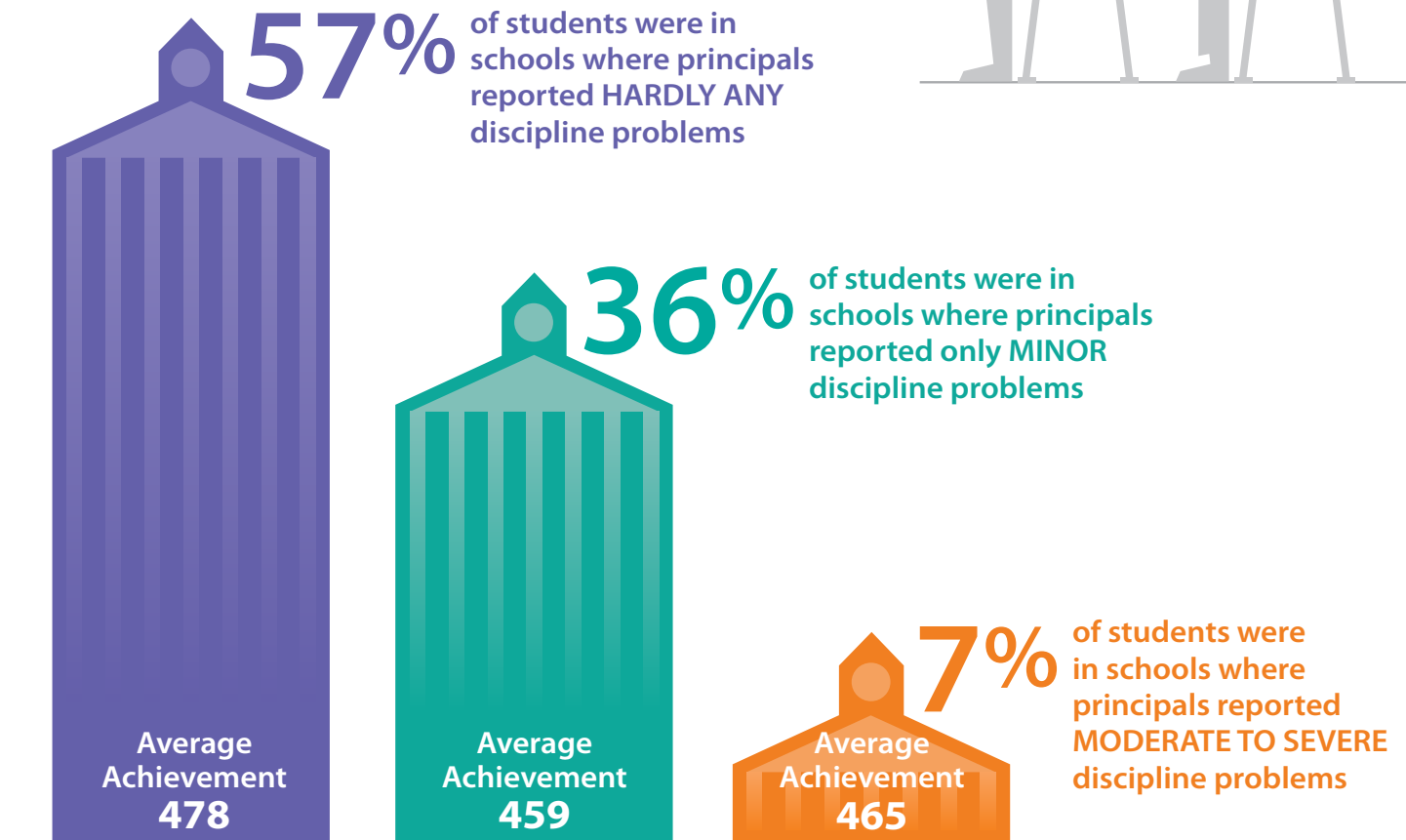


## Students Are in Safe Schools

Principals and teachers agreed that ALMOST ALL the TIMSS Advanced students were in safe school environments.



### Principals' Reports



### Teachers' Reports

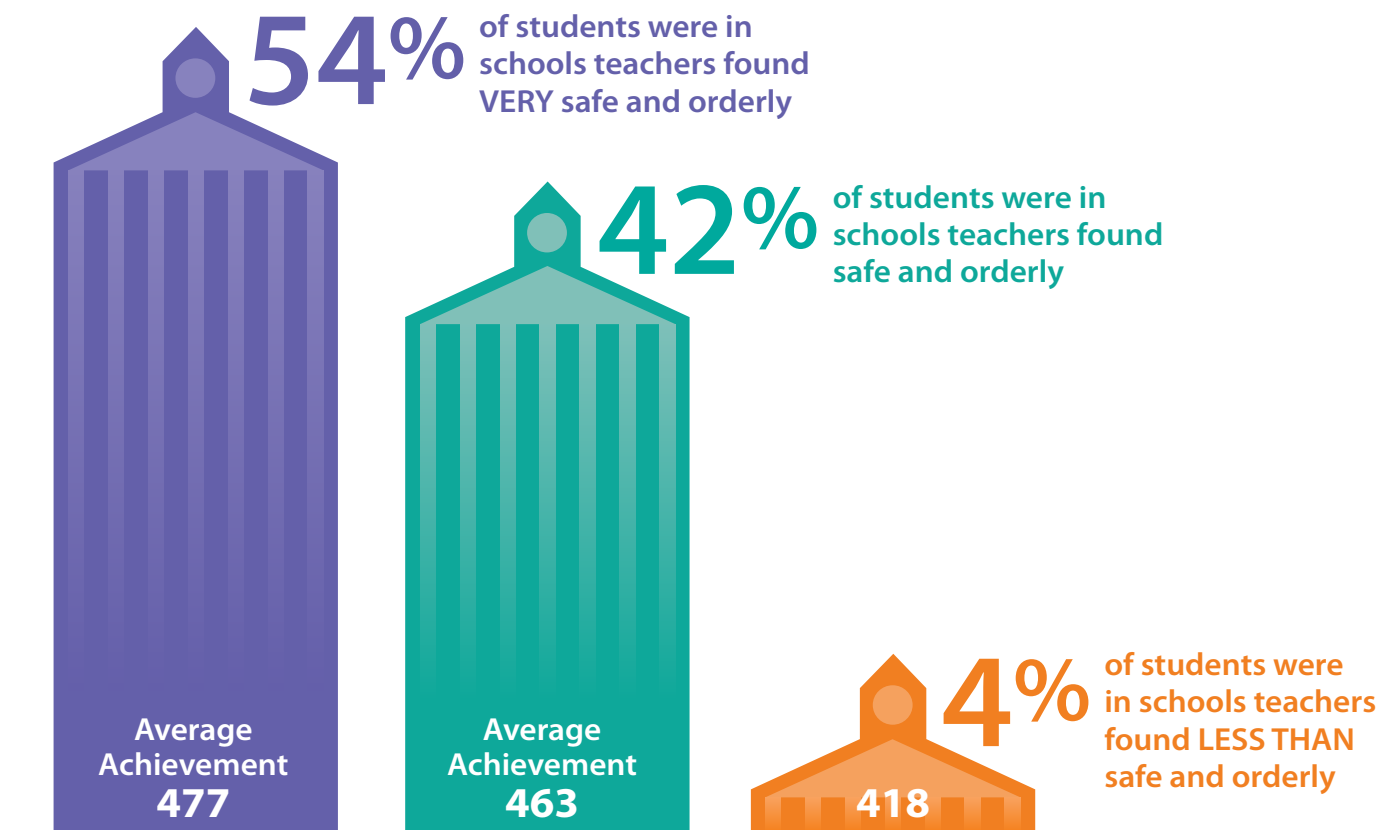




Exhibit M7.1: School Discipline Problems – Principals' Reports

Reported by Principals

Students were scored according to their principals' responses concerning eleven potential school problems on the *School Discipline Problems* scale. Students in schools with **Hardly Any Problems** had a score on the scale of at least 10.0, which corresponds to their principals reporting "not a problem" for six of the eleven issues and "minor problem" for the other five, on average. Students in schools with **Moderate to Severe Problems** had a score no higher than 7.2, which corresponds to their principals reporting "moderate problem" for six of the eleven issues and "minor problem" for the other five, on average. All other students attended schools with **Minor Problems**.

Country	Hardly Any Problems		Minor Problems		Moderate to Severe Problems		Average Scale Score
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	
Russian Federation	85 (2.2)	486 (6.2)	15 (2.2)	474 (14.4)	0 (0.0)	~ ~	11.0 (0.08)
Russian Federation 6hr+	82 (3.5)	538 (9.0)	18 (3.5)	548 (15.7)	0 (0.0)	~ ~	11.0 (0.15)
Norway	66 (8.0)	467 (7.1)	33 (8.0)	453 (6.5)	1 (0.7)	~ ~	10.5 (0.40)
France	65 (4.7)	464 (3.6)	31 (4.5)	455 (6.5)	4 (1.7)	457 (14.5)	10.4 (0.18)
Slovenia	65 (4.7)	469 (5.5)	34 (4.7)	444 (8.3)	1 (0.8)	~ ~	10.3 (0.10)
Portugal	57 (3.9)	483 (3.6)	38 (3.9)	482 (4.8)	6 (1.9)	474 (9.0)	10.2 (0.15)
United States	55 (4.4)	494 (6.2)	43 (4.4)	478 (9.7)	1 (0.7)	~ ~	10.1 (0.14)
Lebanon	48 (3.6)	540 (5.4)	26 (4.8)	522 (4.6)	26 (3.4)	526 (6.0)	9.2 (0.18)
Italy	39 (4.6)	451 (10.4)	39 (4.5)	401 (10.7)	22 (3.5)	403 (18.4)	9.0 (0.20)
Sweden	34 (4.6)	447 (5.8)	65 (4.7)	424 (6.5)	2 (1.0)	~ ~	9.4 (0.11)
International Avg.	57 (1.6)	478 (2.1)	36 (1.6)	459 (2.8)	7 (0.6)	465 (6.4)	

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

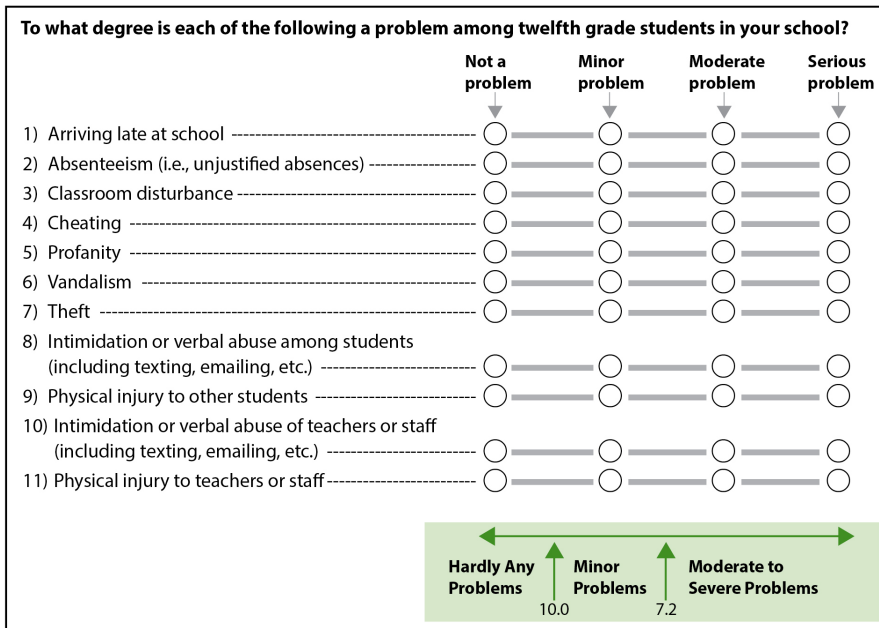
The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

An "r" indicates data are available for at least 70% but less than 85% of the students.





**Exhibit M7.2: Safe and Orderly School – Teachers' Reports**

Reported by Advanced Mathematics Teachers

Students were scored according to their teachers' degree of agreement with eight statements on the *Safe and Orderly School* scale. Students in **Very Safe and Orderly** schools had a score on the scale of at least 9.9, which corresponds to their teachers "agreeing a lot" with four of the eight qualities of a safe and orderly school and "agreeing a little" with the other four, on average. Students in **Less than Safe and Orderly** schools had a score no higher than 6.5, which corresponds to their teachers "disagreeing a little" with four of the eight qualities and "agreeing a little" with the other four, on average. All other students attended **Safe and Orderly** schools.

Country	Very Safe and Orderly		Safe and Orderly		Less than Safe and Orderly		Average Scale Score
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	
Russian Federation 6hr+	78 (4.2)	552 (7.4)	22 (4.2)	499 (19.3)	0 (0.0)	~ ~	10.9 (0.13)
Russian Federation	75 (3.2)	495 (7.0)	24 (3.2)	451 (12.2)	1 (0.6)	~ ~	10.9 (0.13)
Norway	74 (5.1)	465 (5.0)	25 (5.0)	450 (6.8)	1 (1.3)	~ ~	10.8 (0.18)
United States <sup>r</sup>	71 (3.8)	486 (7.4)	26 (3.5)	491 (8.9)	3 (0.9)	408 (20.2)	10.8 (0.19)
Lebanon	67 (4.6)	535 (3.7)	31 (4.7)	526 (7.7)	2 (0.2)	~ ~	10.7 (0.12)
Portugal	52 (3.4)	484 (4.0)	41 (3.7)	481 (3.5)	6 (1.4)	478 (10.7)	9.7 (0.11)
Sweden	44 (4.3)	446 (4.9)	53 (4.3)	427 (6.2)	3 (1.1)	412 (11.1)	9.5 (0.13)
Italy	43 (3.3)	428 (8.0)	51 (3.2)	429 (8.3)	6 (1.8)	355 (27.9)	9.4 (0.14)
France	34 (3.1)	473 (5.6)	61 (3.0)	458 (3.3)	5 (1.4)	426 (13.6)	9.3 (0.15)
Slovenia	27 (3.4)	482 (8.4)	67 (3.1)	454 (4.0)	6 (2.8)	428 (13.4)	8.9 (0.17)
International Avg.	54 (1.3)	477 (2.1)	42 (1.3)	463 (2.4)	4 (0.5)	418 (7.1)	

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

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( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

An "r" indicates data are available for at least 70% but less than 85% of the students.

**Thinking about your current school, indicate the extent to which you agree or disagree with each of the following statements.**

Agree a lot    Agree a little    Disagree a little    Disagree a lot

- 1) This school is located in a safe neighborhood ----- ○ ———— ○ ———— ○ ———— ○
- 2) I feel safe at this school ----- ○ ———— ○ ———— ○ ———— ○
- 3) This school's security policies and practices are sufficient ----- ○ ———— ○ ———— ○ ———— ○
- 4) The students behave in an orderly manner ----- ○ ———— ○ ———— ○ ———— ○
- 5) The students are respectful of the teachers ----- ○ ———— ○ ———— ○ ———— ○
- 6) The students respect school property ----- ○ ———— ○ ———— ○ ———— ○
- 7) This school has clear rules about student conduct ----- ○ ———— ○ ———— ○ ———— ○
- 8) This school's rules are enforced in a fair and consistent manner ----- ○ ———— ○ ———— ○ ———— ○

Very Safe and Orderly    Safe and Orderly    Less than Safe and Orderly

9.9    6.5



# CHAPTER M8: TEACHERS' AND PRINCIPALS' PREPARATION

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS



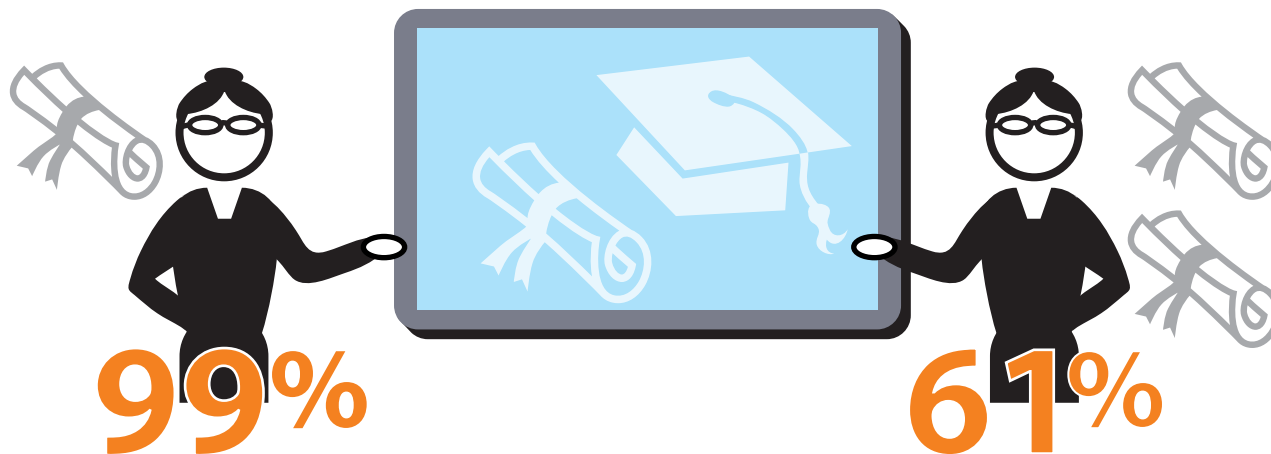
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## Students Have Well Qualified Teachers and Principals

Mathematics teachers of TIMSS Advanced students reported high levels of education and considerable experience.



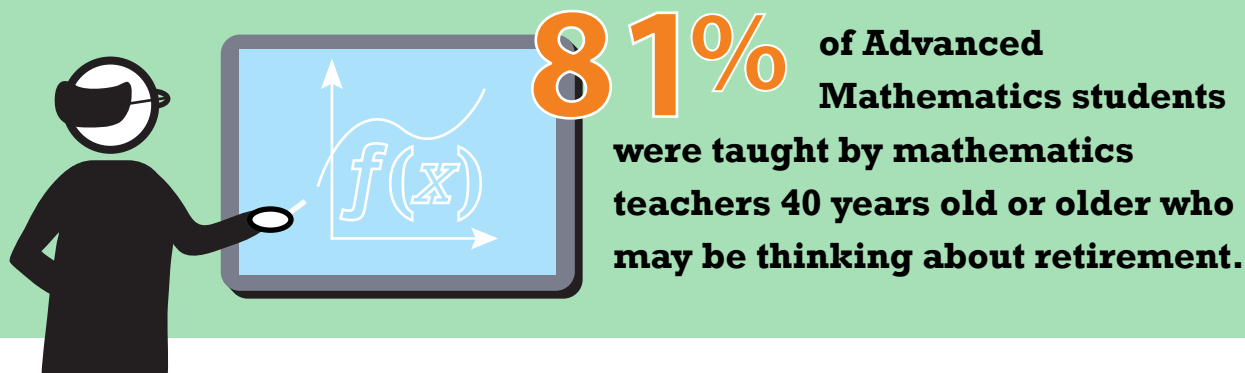
**99%** of students were taught by mathematics teachers with at least a Bachelor's degree

**61%** of students were taught by mathematics teachers with a Master's or Doctorate degree



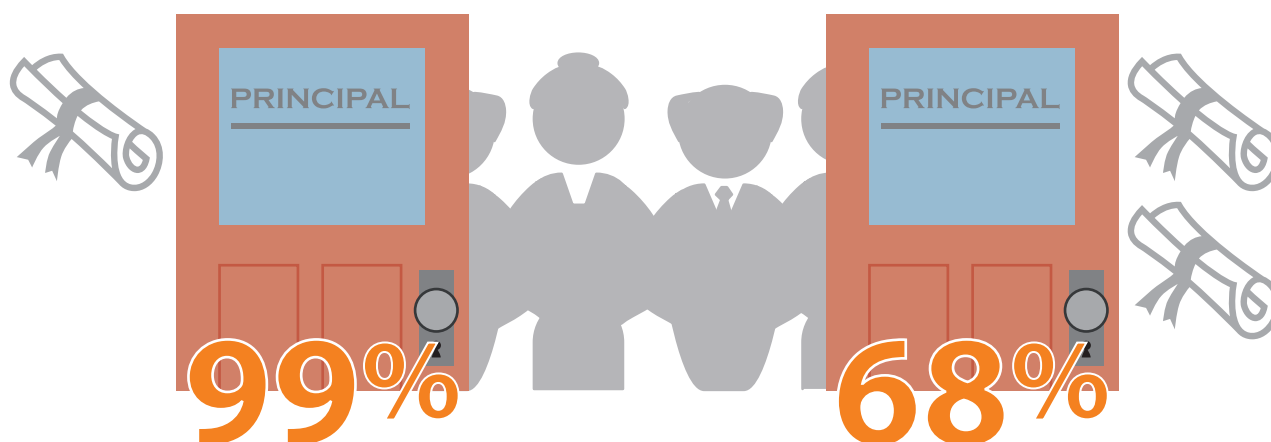
On average, students were taught by mathematics teachers with 14 years of experience teaching advanced mathematics.

The TIMSS Advanced countries have special requirements for teaching advanced mathematics, so it may be a concern that:



**81%** of Advanced Mathematics students were taught by mathematics teachers 40 years old or older who may be thinking about retirement.

Principals of schools with TIMSS Advanced Mathematics students reported high levels of education and experience.



**99%** of students had principals with at least a Bachelor's degree

**68%** of students had principals with a Master's or Doctorate degree

On average, principals had 11 years of experience.



**Exhibit M8.1: National Requirements for Being an Advanced Mathematics Teacher in the Final Year of School**

*Reported by National Research Coordinators*

Country	Requirements
France	Teachers must be qualified to teach secondary school. Secondary school teachers must hold a master's degree and pass the competitive national examination.
Italy	Teachers must be officially qualified to teach mathematics in upper-secondary school (graduate in mathematics or physics, statistics, etc. and have a certification for teaching mathematics, released by a university after a 1-year course for teaching qualification).
Lebanon	Teachers must hold a bachelor's degree in mathematics, as well as a teaching diploma.
Norway	Teachers are required to have at least a university bachelor's degree and have taken at least 1 full year (60 credit points) of mathematics courses. They also need 1 year of teacher education courses, consisting of general pedagogy, mathematics education, and teaching practice in schools. These courses may be taken separately after finishing subject studies, or as an integrated part. The current tendency is that a full master's degree will be required.
Portugal	Fully qualified advanced mathematics teachers must have at least a Master of Science degree in mathematical education, which includes a professional internship. They must pass both a general knowledge and a specific (mathematics) teachers' qualifying examination.
Russian Federation	Teachers must have a university degree in mathematics education, or any university degree and certificate of additional courses in mathematics education. There are no official requirements for being a teacher of advanced mathematics. Beginning in 2017, according to the new professional standards, teachers of advanced mathematics should have at least a master's degree.
Slovenia	All teachers must have an appropriate university degree, pedagogical training, and have successfully completed the teaching certification examination. Teachers for the advanced mathematics program should have a second-level university degree, which means 5 years of mathematics university study, including pedagogical courses.
Sweden	Teachers must be licensed through a teacher education program. To become a mathematics teacher in upper-secondary school you need at least 1.5 years of mathematics courses at a university. You also need 1.5–2 years of tertiary level studies in one more subject, and 1.5 years of courses in specific and general education. In total, 300 credits for 5–5.5 years are required. After finishing a teacher education program, prospective teachers apply for a license at the Swedish National Agency for Education.
United States	All public school teachers must be licensed by their state's department of education, and requirements for licensure vary by state. Secondary school mathematics teachers may have a bachelor's degree in mathematics (and possibly a master's degree in education), or a double major in mathematics and education. Additionally, all teachers must be highly qualified, which includes demonstrating of expertise in their subject area by either passing a subject test or completing an undergraduate degree, completing a graduate degree, completing coursework equivalent to an undergraduate major, or completing advanced certification or credentialing.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M8.2: Advanced Mathematics Teachers' Formal Education\***

Reported by Advanced Mathematics Teachers

Country	Completed Postgraduate University Degree**		Completed Bachelor's Degree or Equivalent but Not a Postgraduate Degree		Did Not Complete Bachelor's Degree	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
France	75 (2.9)	462 (3.3)	25 (2.9)	460 (4.7)	0 (0.0)	~ ~
Italy	11 (2.0)	388 (18.2)	87 (2.2)	428 (6.4)	2 (0.9)	~ ~
Lebanon	59 (4.1)	529 (3.6)	34 (4.2)	542 (7.3)	7 (1.5)	524 (12.6)
Norway	77 (3.2)	464 (4.7)	22 (3.2)	452 (9.4)	0 (0.0)	~ ~
Portugal	20 (2.8)	483 (5.4)	78 (2.9)	482 (2.9)	2 (0.8)	~ ~
Russian Federation	71 (3.3)	483 (8.0)	29 (3.3)	485 (12.0)	0 (0.0)	~ ~
Russian Federation 6hr+	81 (2.7)	547 (9.2)	19 (2.7)	513 (13.4)	0 (0.0)	~ ~
Slovenia	99 (0.8)	460 (3.5)	1 (0.8)	~ ~	0 (0.0)	~ ~
Sweden	67 (3.5)	436 (4.8)	32 (3.4)	437 (7.2)	2 (0.6)	~ ~
United States <sup>r</sup>	73 (4.3)	493 (5.4)	27 (4.3)	464 (13.2)	0 (0.0)	~ ~
International Avg.	61 (1.1)	466 (2.6)	37 (1.1)	469 (3.0)	1 (0.2)	524 (12.6)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

\* Based on countries' categorizations according to UNESCO's International Standard Classification of Education (Operational Manual for ISCED-2011).

\*\* For example, doctorate, master's, or other postgraduate degree.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

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### Exhibit M8.3: Advanced Mathematics Teachers Majored in Mathematics and Education

Reported by Advanced Mathematics Teachers

Country	Major in Mathematics and Mathematics Education		Major in Mathematics but No Major in Mathematics Education		Major in Mathematics Education but No Major in Mathematics		All Other Majors	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
France	32 (2.7)	465 (4.1)	68 (2.7)	460 (3.9)	0 (0.4)	~ ~	0 (0.2)	~ ~
Italy	49 (4.1)	417 (10.9)	41 (3.8)	444 (8.7)	1 (0.6)	~ ~	9 (1.9)	379 (17.6)
Lebanon	61 (4.1)	534 (4.6)	33 (2.5)	538 (5.1)	1 (0.6)	~ ~	5 (4.2)	514 (14.0)
Norway	18 (3.5)	466 (11.4)	80 (3.7)	461 (4.0)	0 (0.0)	~ ~	2 (1.3)	~ ~
Portugal	76 (3.1)	480 (3.4)	19 (2.6)	489 (5.3)	3 (1.2)	496 (8.1)	2 (1.0)	~ ~
Russian Federation	60 (3.8)	492 (7.7)	39 (3.8)	468 (7.7)	0 (0.0)	~ ~	1 (0.5)	~ ~
Russian Federation 6hr+	69 (5.1)	555 (8.3)	30 (5.0)	505 (16.6)	0 (0.0)	~ ~	1 (0.5)	~ ~
Slovenia	56 (4.3)	465 (4.7)	29 (3.5)	452 (8.4)	13 (2.6)	458 (7.1)	2 (0.6)	~ ~
Sweden	71 (3.7)	440 (4.8)	20 (3.0)	426 (7.2)	7 (2.0)	437 (9.5)	2 (0.9)	~ ~
United States <sub>r</sub>	58 (3.2)	478 (8.2)	22 (2.7)	500 (8.6)	11 (1.9)	497 (13.6)	9 (1.0)	482 (8.7)
International Avg.	53 (1.2)	471 (2.4)	39 (1.1)	471 (2.3)	4 (0.5)	472 (4.9)	4 (0.6)	458 (8.0)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

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**Exhibit M8.4: Advanced Mathematics Teachers' Gender, Age, and Number of Years Teaching**

Reported by Advanced Mathematics Teachers

Country	Percent of Students by Teacher Characteristics						Average Number of Years Teaching		
	Gender		Age					Teaching Altogether	Teaching Mathematics at the Advanced Level
	Female	Male	29 Years or Under	30-39 Years	40-49 Years	50-59 Years	60 Years or Older		
France	40 (3.2)	60 (3.2)	4 (1.2)	17 (2.3)	43 (3.2)	30 (2.7)	6 (1.4)	23 (0.5)	9 (0.4)
Italy	67 (3.6)	33 (3.6)	0 (0.0)	2 (0.7)	31 (3.4)	50 (3.7)	17 (2.6)	25 (0.5)	17 (0.6)
Lebanon	18 (1.7)	82 (1.7)	4 (1.4)	21 (2.3)	29 (4.9)	23 (3.7)	23 (2.3)	25 (0.6)	20 (0.7)
Norway	25 (4.5)	75 (4.5)	3 (1.5)	24 (4.9)	25 (3.3)	20 (3.1)	29 (5.9)	20 (1.6)	13 (1.3)
Portugal	75 (2.8)	25 (2.8)	0 (0.0)	12 (2.6)	44 (3.4)	39 (3.3)	6 (1.6)	25 (0.5)	10 (0.4)
Russian Federation	96 (1.2)	4 (1.2)	1 (0.7)	7 (1.5)	35 (3.4)	39 (3.2)	17 (2.8)	28 (0.6)	9 (0.5)
Russian Federation 6hr+	91 (2.1)	9 (2.1)	2 (1.1)	5 (1.6)	39 (4.4)	41 (4.8)	14 (2.9)	28 (0.7)	14 (0.7)
Slovenia	75 (3.1)	25 (3.1)	1 (0.8)	24 (3.8)	35 (3.9)	34 (4.1)	6 (1.4)	22 (0.8)	18 (0.5)
Sweden	30 (4.4)	70 (4.4)	4 (1.7)	20 (3.0)	30 (4.1)	21 (3.4)	24 (2.6)	18 (1.0)	13 (0.9)
United States	r 44 (4.0)	56 (4.0)	r 7 (2.2)	21 (2.7)	37 (3.1)	19 (2.2)	15 (3.6)	r 20 (0.9)	r 13 (0.8)
International Avg.	52 (1.1)	48 (1.1)	3 (0.4)	17 (1.0)	34 (1.2)	31 (1.1)	16 (1.0)	23 (0.3)	14 (0.2)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding, some results may appear inconsistent. An "r" indicates data are available for at least 70% but less than 85% of the students.

**Exhibit M8.5: Advanced Mathematics Teachers' Participation in Professional Development in Mathematics in the Past Two Years**

*Reported by Advanced Mathematics Teachers*

Teachers could indicate participating in more than one area of professional development.

Country	Percent of Students by Teacher's Area of Professional Development						
	Mathematics Content	Mathematics Pedagogy/ Instruction	Mathematics Curriculum	Integrating Information Technology into Mathematics	Improving Students' Critical Thinking or Problem Solving Skills	Mathematics Assessment	Addressing Individual Students' Needs
France	33 (3.2)	36 (3.3)	27 (3.0)	32 (3.2)	12 (2.0)	15 (2.9)	12 (2.1)
Italy	41 (4.0)	50 (3.7)	28 (3.3)	48 (3.6)	12 (2.4)	17 (3.1)	19 (3.3)
Lebanon	47 (4.8)	54 (3.4)	41 (4.2)	55 (3.2)	49 (3.4)	50 (3.4)	42 (2.8)
Norway	18 (4.3)	20 (4.3)	26 (3.8)	49 (4.5)	5 (2.3)	28 (4.4)	9 (3.2)
Portugal	77 (2.9)	55 (3.3)	70 (3.2)	56 (3.5)	14 (2.5)	33 (3.1)	10 (2.0)
Russian Federation	73 (2.6)	79 (2.3)	78 (2.4)	71 (2.6)	42 (3.5)	56 (3.2)	49 (3.5)
Russian Federation 6hr+	65 (4.7)	77 (4.4)	68 (4.7)	66 (5.0)	41 (4.5)	54 (5.3)	46 (4.3)
Slovenia	72 (3.0)	39 (4.3)	24 (3.7)	65 (3.2)	38 (3.1)	22 (3.5)	17 (2.8)
Sweden	41 (3.9)	67 (4.0)	38 (4.2)	37 (5.3)	41 (4.4)	66 (4.0)	19 (2.3)
United States	r 66 (3.3)	r 65 (3.4)	r 70 (3.7)	r 56 (3.3)	r 56 (4.2)	r 47 (3.7)	r 53 (4.2)
International Avg.	52 (1.2)	52 (1.2)	45 (1.2)	52 (1.2)	30 (1.1)	37 (1.2)	25 (1.0)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

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An "r" indicates data are available for at least 70% but less than 85% of the students.

**Exhibit M8.6: Principals' Formal Education\***

Reported by Principals

Country	Percent of Students by Principal Educational Level		
	Completed Postgraduate University Degree**	Completed Bachelor's Degree or Equivalent but Not a Postgraduate Degree	Did Not Complete Bachelor's Degree
France	73 (4.0)	25 (4.0)	1 (1.1)
Italy	27 (4.5)	69 (4.4)	4 (2.0)
Lebanon	71 (4.3)	27 (4.2)	3 (0.5)
Norway <sub>r</sub>	70 (6.1)	30 (6.1)	0 (0.0)
Portugal	38 (4.0)	62 (4.0)	0 (0.0)
Russian Federation	85 (2.5)	15 (2.5)	0 (0.0)
Russian Federation 6hr+	91 (3.8)	9 (3.8)	0 (0.0)
Slovenia	100 (0.0)	0 (0.0)	0 (0.0)
Sweden	49 (5.1)	47 (5.1)	4 (1.5)
United States	99 (0.7)	1 (0.7)	0 (0.0)
International Avg.	68 (1.3)	31 (1.3)	1 (0.3)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

\* Based on countries' categorizations according to UNESCO's International Standard Classification of Education (Operational Manual for ISCED-2011).

\*\* For example, doctorate, master's, or other postgraduate degree.

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An "r" indicates data are available for at least 70% but less than 85% of the students.

**Exhibit M8.7: Principals' Years of Experience**

Reported by Principals

Country	Percent of Students by Principal Years of Experience as a Principal				Average Years of Experience as a Principal
	20 Years or More	At Least 10 but Less than 20 Years	At Least 5 but Less than 10 Years	Less than 5 Years	
France	14 (2.7)	31 (4.1)	27 (3.9)	28 (4.4)	10 (0.7)
Italy	17 (3.6)	28 (4.5)	40 (4.9)	15 (3.4)	11 (0.7)
Lebanon	33 (5.1)	20 (2.7)	26 (3.5)	21 (3.0)	14 (0.9)
Norway	37 (9.0)	33 (8.1)	22 (5.4)	9 (4.0)	16 (1.7)
Portugal	17 (2.6)	33 (3.4)	27 (3.4)	22 (3.3)	11 (0.6)
Russian Federation	20 (2.5)	31 (3.6)	30 (3.1)	18 (1.8)	12 (0.5)
Russian Federation 6hr+	25 (3.9)	35 (4.8)	26 (5.4)	14 (2.6)	14 (0.9)
Slovenia	12 (3.6)	33 (3.6)	28 (2.5)	27 (2.1)	11 (0.6)
Sweden	5 (2.5)	29 (3.8)	42 (4.6)	24 (4.1)	9 (0.5)
United States	10 (2.7)	30 (4.6)	29 (4.3)	32 (5.1)	9 (0.7)
International Avg.	18 (1.4)	30 (1.5)	30 (1.4)	22 (1.2)	11 (0.3)

SOURCE: IEA's Trends in International Mathematics and Science Study - TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

An "r" indicates data are available for at least 70% but less than 85% of the students.





# CHAPTER M9: CLASSROOM INSTRUCTION

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS



**IEA**

**TIMSS & PIRLS**  
International Study Center  
Lynch School of Education, Boston College

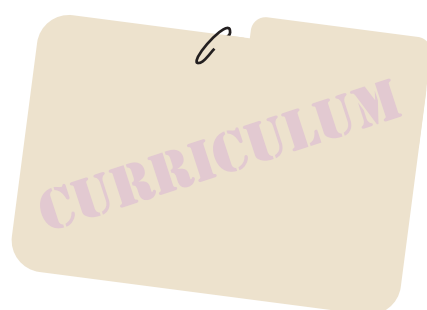


## Instruction in Advanced Mathematics Classes

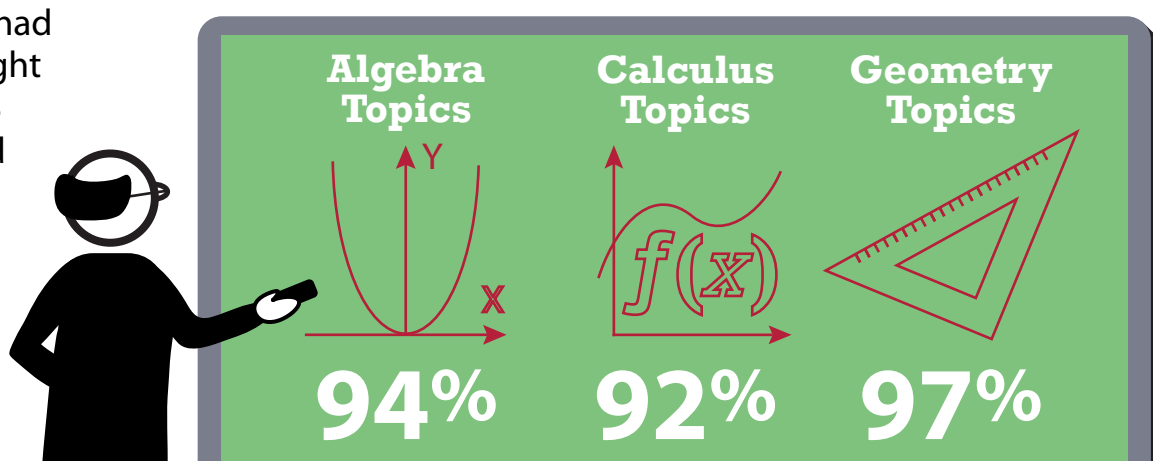
### Curriculum

Covering a rigorous curriculum is key in students' opportunity to learn.

Eight of the nine countries participating in TIMSS Advanced had a national curriculum, with the United States being the exception. All but two (Sweden and the United States) had a "high stakes" test for students nearing the completion of secondary school.



There was variation in topic coverage within content domains. However, according to their teachers, on average, most Advanced Mathematics students had been taught the TIMSS Advanced topics.

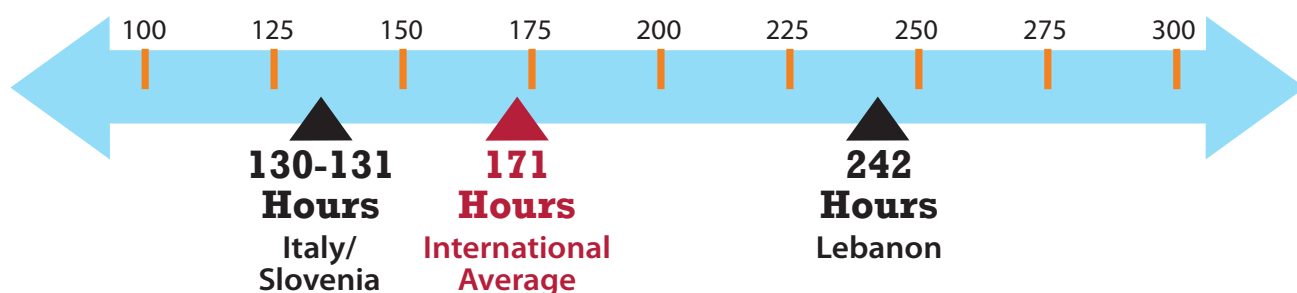


### Instructional Time

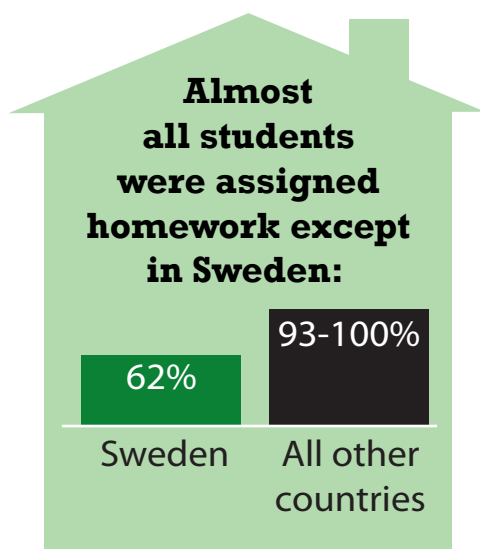
Instructional time remains a crucial resource in considering students' opportunity to learn in their final year, even though there are many factors that influence the effectiveness of an educational system.



There was a considerable range in the yearly number of instructional hours in advanced mathematics.



Students also studied outside of school:

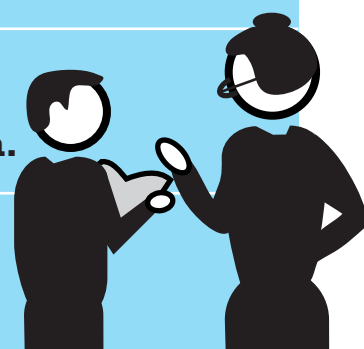


Large percentages of Advanced Mathematics students reported attending extra tutoring outside of school to improve their achievement.

**3/5** attended tutoring in Portugal and the Russian Federation.

**1/3** attended tutoring in France, Italy, and Slovenia.

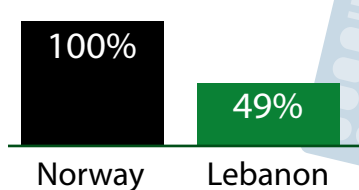
On average, students attending extra tutoring had lower achievement.



### Technology

There is a continuing debate about the role of technology in education, and more particularly in mathematics classes.

Across the TIMSS Advanced countries there was a wide range in access to digital devices to use in advanced mathematics lessons, with 78% of students on average having digital devices available.



Teachers have students use their digital devices primarily to draw graphs of functions (68%) and solve equations (63%).

Students used the Internet for their TIMSS Advanced school work primarily to:

Find information about mathematics concepts or solve problems

Access course materials and do homework



68-70%

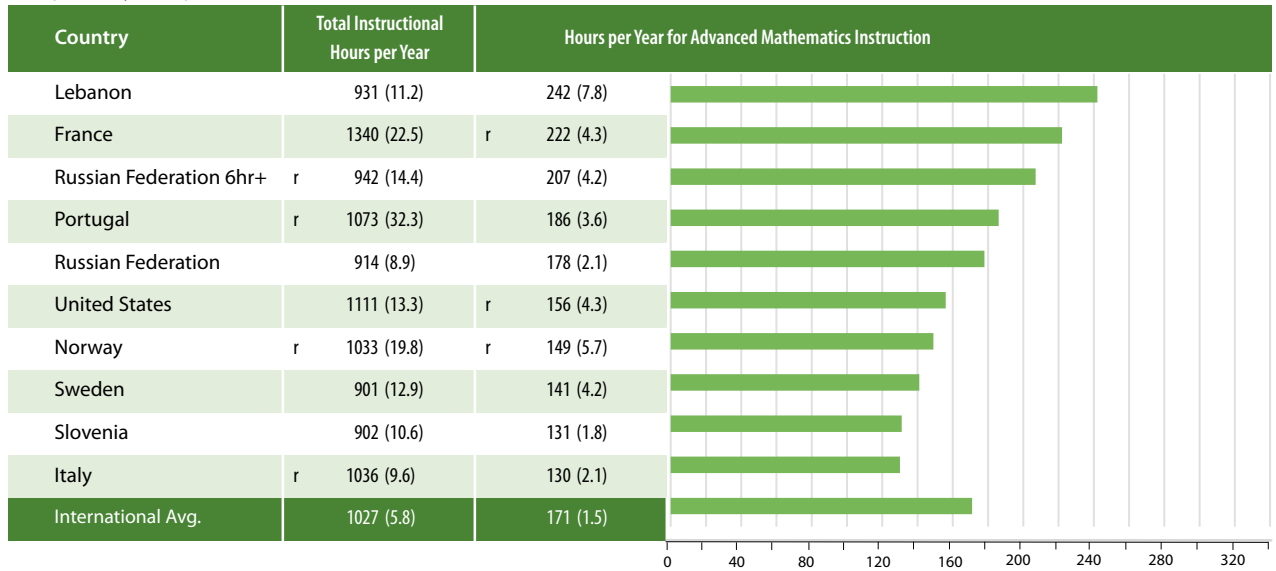
50-54%





**Exhibit M9.1: Instructional Time Spent on Advanced Mathematics**

Reported by Principals and Advanced Mathematics Teachers



SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS-Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

An "r" indicates data are available for at least 70% but less than 85% of the students.

<b>Total Instructional Hours per Year</b>	=	Principal Reports of School Days per Year	x	Principal Reports of Instructional Hours per Day
<b>Hours per Year for Advanced Mathematics Instruction</b>	=	Teacher Reports of Weekly Advanced Mathematics Instructional Hours	x	Principal Reports of School Days per Year
		Principal Reports of School Days per Week		

**Exhibit M9.2: Types of Homework Assignments**

Reported by Advanced Mathematics Teachers

Country	Mathematics Homework Assigned to Class			Percent of Students Whose Teachers "Sometimes" or "Always or Almost Always" Assign Each Type of Homework					
	Percent of Students	Average Achievement		Doing Problem/Question Sets	Reading the Textbook	Memorizing Formulas and Procedures	Gathering, Analyzing, and Reporting Data	Finding Applications of the Content Covered	Working on Projects
	Yes	Yes	No						
France	100 (0.0)	461 (3.1)	~ ~	100 (0.3)	42 (3.1)	90 (1.9)	68 (2.6)	43 (3.3)	20 (2.3)
Italy	98 (0.9)	424 (5.7)	~ ~	98 (1.0)	80 (3.0)	71 (2.8)	58 (3.3)	73 (3.2)	27 (2.8)
Lebanon	93 (1.9)	531 (3.2)	543 (13.8)	93 (1.9)	79 (2.4)	84 (2.2)	83 (2.7)	84 (2.4)	56 (4.8)
Norway	94 (2.4)	461 (4.7)	477 (18.2)	93 (2.6)	73 (4.4)	69 (4.5)	19 (3.5)	29 (4.4)	5 (2.1)
Portugal	96 (1.6)	484 (2.6)	468 (11.3)	94 (1.9)	63 (3.3)	54 (3.8)	44 (4.1)	58 (3.6)	13 (2.0)
Russian Federation	100 (0.0)	484 (5.8)	~ ~	100 (0.0)	95 (1.5)	96 (1.3)	89 (2.0)	93 (1.5)	73 (3.3)
Russian Federation 6hr+	100 (0.0)	540 (8.1)	~ ~	100 (0.0)	92 (1.7)	91 (2.7)	84 (2.5)	89 (2.7)	72 (4.3)
Slovenia	97 (1.0)	460 (3.7)	465 (18.0)	97 (1.0)	38 (3.0)	76 (3.1)	33 (2.3)	29 (3.5)	21 (2.5)
Sweden	62 (4.4)	431 (5.4)	441 (6.2)	60 (4.4)	38 (3.4)	22 (3.0)	15 (2.9)	24 (3.3)	18 (2.5)
United States	r 98 (1.4)	485 (5.8)	~ ~	r 98 (1.4)	r 58 (3.5)	r 78 (2.5)	r 52 (4.1)	r 65 (4.1)	r 63 (3.6)
International Avg.	93 (0.7)	469 (1.5)	479 (6.4)	92 (0.7)	63 (1.1)	71 (1.0)	51 (1.1)	55 (1.1)	33 (1.0)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

An "r" indicates data are available for at least 70% but less than 85% of the students.

**Exhibit M9.3: Students Attended Extra Tutoring in Advanced Mathematics Not Provided by the School**

Reported by Advanced Mathematics Students

Country	Students Did Not Attend Extra Tutoring		Students Attended Extra Tutoring		Reasons for Attending Extra Tutoring (Students Could Indicate More than One)					
					To Excel in Class		To Keep Up in Class		To Do Well on an Examination	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
France	65 (1.0)	476 (3.2)	35 (1.0)	438 (3.6)	10 (0.6)	468 (5.2)	23 (0.9)	426 (3.5)	28 (0.8)	436 (3.5)
Italy	67 (1.2)	434 (5.7)	33 (1.2)	397 (6.4)	5 (0.4)	414 (11.0)	23 (0.9)	383 (6.4)	18 (0.9)	402 (6.9)
Lebanon	84 (1.4)	540 (3.0)	16 (1.4)	494 (5.5)	7 (0.8)	501 (9.3)	5 (0.6)	472 (8.4)	10 (1.1)	486 (6.0)
Norway	93 (0.8)	462 (4.6)	7 (0.8)	428 (7.8)	4 (0.8)	432 (10.7)	4 (0.5)	409 (10.6)	5 (0.6)	429 (8.7)
Portugal	39 (1.5)	491 (3.4)	61 (1.5)	477 (2.6)	38 (1.3)	484 (3.3)	46 (1.4)	466 (2.8)	54 (1.6)	478 (2.7)
Russian Federation	33 (1.3)	491 (7.3)	67 (1.3)	482 (5.5)	23 (1.1)	488 (6.6)	18 (0.9)	461 (8.1)	64 (1.4)	481 (5.4)
Russian Federation 6hr+	38 (2.8)	553 (8.6)	62 (2.8)	533 (8.7)	21 (1.8)	533 (11.5)	15 (1.4)	500 (11.4)	60 (2.7)	532 (9.0)
Slovenia	70 (1.2)	481 (3.3)	30 (1.2)	414 (5.1)	11 (0.9)	424 (7.9)	17 (0.8)	396 (5.8)	25 (1.1)	410 (5.1)
Sweden	89 (0.7)	438 (4.0)	11 (0.7)	379 (7.5)	6 (0.4)	397 (7.1)	5 (0.4)	347 (9.4)	9 (0.6)	371 (7.1)
United States	88 (0.9)	489 (5.4)	12 (0.9)	462 (7.6)	8 (0.7)	463 (10.9)	9 (0.7)	448 (7.9)	10 (0.8)	462 (8.8)
International Avg.	70 (0.4)	478 (1.5)	30 (0.4)	441 (2.0)	12 (0.3)	452 (2.8)	17 (0.3)	423 (2.5)	25 (0.3)	439 (2.1)

SOURCE: IEA's Trends in International Mathematics and Science Study - TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M9.4: Examinations with Consequences for Students in Advanced Mathematics Programs (Tracks)**

Reported by National Research Coordinators

Country	Examinations with Consequences for Individual Students	Grades at Which Examinations with Consequences for Individual Students are Given	Format of Examinations with Consequences for Individual Students	Comments
France	●	Grade 12	Written and/or oral examinations	Each subject examination grade is weighted differently according to the track students are attending. In the scientific track, mathematics and other science grades altogether are weighted as much as half the student's total grade average.
Italy	●	Grades 8 and 13	Written and oral examinations	Final examinations for technical and vocational secondary schools (Grade 13) also give students an opportunity to find a job.
Lebanon	●	Grade 12	Written examinations	At the end of the third year of the secondary cycle or Grade 12, students have to pass the Official Baccalaureate exams for four sections—life sciences, general sciences, economics, and humanities. The purpose of these exams is for the students to be able to continue with their university studies.
Norway	●	Grades 12 and 13	Written or oral examinations	A written examination is set and marked centrally (at national level) and an oral examination is prepared and marked locally. About 40% of the first year (Mathematics R1) students and about 60% of the second year (Mathematics R2) students are sampled for the national written examination. For the local oral examination, about 5% and 15% of the students in the respective courses are sampled for testing.
Portugal	●	Grades 9 and 12	Written examinations	Nationwide final examinations are produced by an independent educational assessment public institute (IAVE, I. P.). The application and scoring of the examinations is coordinated by a National Exam Jury Board under the supervision of the General Education Directorate of the Ministry of Education.
Russian Federation	●	Grade 11	Written examinations	Tests for the compulsory state examination in Grade 11 are given at both the Basic and Profile levels, and all graduates are offered a choice to take one of these exams, no matter what course they studied in Grades 10 and 11.
Slovenia	●	End of upper-secondary education	There are several examination formats—written only; both written and oral; both written and practical; written, oral and practical; practical only; and an examination presentation.	Achievement on the Matura examination and achievement in the last two years of schooling are used to select students where there is a limit to the number of candidates for a university program. The Matura is prepared and administered by the National Examination Center.
Sweden	○	n/a	n/a	Compulsory national tests are developed by the Swedish National Agency for Education, which is the educational authority appointed by the National Ministry of Education for the administration of the school system. These national tests do not have direct consequences for the students because they are intended only to support teachers' assessment of students.
United States	○	n/a	n/a	Although there are no national exams with consequences for individual students, many high school students take Advanced Placement (AP) or International Baccalaureate (IB) courses that culminate with an end-of-course exam. Students can take these AP or IB written exams at a price and, if they score well, can earn course credit at many colleges and universities. In addition, to apply for admission to most colleges and universities in the United States, students in Grades 11 and 12 take written exams to demonstrate their readiness for college-level work. Private companies (e.g., ACT, College Board) offer these exams in different subjects to students for a price.

● Yes  
○ No

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M9.5: Characteristics and Methods Used to Evaluate the Advanced Mathematics Curriculum**

Reported by National Research Coordinators

Country	National Curriculum	Year Introduced	Being Revised	Methods Used to Evaluate the Implementation of the Advanced Mathematics Curriculum			
				Visits by Inspectors	Research Programs	School Self-Evaluation	National or Regional Assessments
France	●	2011	○	●	○	○	●
Italy	●	2010	○	○	○	●	●
Lebanon	●	2001	○	●	○	●	●
Norway	●	2006	○	○	○	○	●
Portugal	●	2003	●	○	○	●	●
Russian Federation	●	2004	●	○	●	●	●
Slovenia	●	2008	○	○	●	●	●
Sweden	●	2011	○	●	●	●	●
United States	○	Varies by school and by course	●	Varies by state	●	●	○

● Yes  
○ No

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M9.6: Number of TIMSS Advanced Advanced Mathematics Topics in the Intended Curriculum**

Reported by National Research Coordinators

Country	All Advanced Mathematics (19 topics)	Algebra (8 topics)	Calculus (7 topics)	Geometry (4 topics)
France	18	8	6	4
Italy	19	8	7	4
Lebanon	19	8	7	4
Norway	18	7	7	4
Portugal	18	8	6	4
Russian Federation	16	8	4	4
Russian Federation 6hr+	19	8	7	4
Slovenia	19	8	7	4
Sweden	17	8	5	4
United States	19	8	7	4

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

In the United States, the number of TIMSS Advanced mathematics topics covered varies by state and course type. The data shown in this table reflect the maximum number of topics that may be covered in each content domain.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS-Advanced 2015

**TIMSS Advanced 2015 Advanced Mathematics Topics**

**A. Algebra**

- 1) Operations with exponential, logarithmic, polynomial, rational, and radical expressions
- 2) Operations with complex numbers
- 3) Evaluating algebraic expressions
- 4) The  $n$ th term of arithmetic and geometric sequences and the sums of finite and infinite series
- 5) Linear, simultaneous, and quadratic equations and inequalities; radical expressions, logarithmic, and exponential equations
- 6) Slopes,  $y$ -axis intercepts, and points of intersection of straight lines
- 7) Equivalent representations of functions, including composite functions, as ordered pairs, tables, graphs, formulas, or words
- 8) Properties of functions including domain and range

**B. Calculus**

- 1) Limits of functions
- 2) Conditions for continuity and differentiability of functions
- 3) Differentiation of functions; differentiation of products, quotients, and composite functions
- 4) Using derivatives to solve problems
- 5) Using first and second derivatives to determine slope and local extrema of functions
- 6) Using derivatives to determine points of inflection of functions
- 7) Integrating functions; evaluating definite integrals, including calculation of areas

**C. Geometry**

- 1) Properties of geometric figures in two and three dimensions
- 2) Properties of vectors and their sums and differences
- 3) Trigonometric properties of triangles (sine, cosine, and tangent)
- 4) Trigonometric functions and their graphs

**Exhibit M9.7: Percentages of Students Taught\* the TIMSS Advanced Topics in Algebra, Calculus, and Geometry**

**Algebra Topics**

Reported by Advanced Mathematics Teachers

Country	Algebra Topics							
	Operations with Expressions	Operations with Complex Numbers	Evaluating Algebraic Expressions	Sequences and Series	Equations and Inequalities	Straight Lines	Representations of Functions	Properties of Functions
France	99 (0.4)	99 (0.4)	98 (0.5)	97 (1.0)	99 (0.4)	99 (0.4)	96 (1.1)	98 (0.7)
Italy	100 (0.0)	71 (3.9)	99 (0.6)	48 (3.2)	100 (0.0)	100 (0.0)	94 (1.7)	99 (0.8)
Lebanon	99 (0.1)	100 (0.1)	99 (0.1)	98 (0.6)	100 (0.0)	98 (0.8)	93 (1.7)	97 (1.2)
Norway	98 (1.5)	8 (3.1)	98 (1.5)	80 (3.8)	99 (0.4)	98 (1.5)	96 (1.8)	98 (1.5)
Portugal	100 (0.0)	74 (3.7)	100 (0.0)	68 (3.2)	100 (0.0)	100 (0.0)	100 (0.0)	100 (0.0)
Russian Federation	--	--	--	--	--	--	--	--
Russian Federation 6hr+	--	--	--	--	--	--	--	--
Slovenia	100 (0.5)	100 (0.5)	100 (0.5)	100 (0.5)	100 (0.5)	100 (0.0)	96 (1.5)	100 (0.0)
Sweden	100 (0.0)	76 (4.4)	100 (0.3)	45 (4.3)	100 (0.0)	100 (0.0)	100 (0.3)	100 (0.0)
United States	r 100 (0.3)	r 98 (1.3)	r 100 (0.3)	r 88 (2.8)	r 100 (0.3)	r 100 (0.0)	r 100 (0.0)	r 100 (0.0)
International Avg.	99 (0.2)	78 (1.0)	99 (0.2)	78 (1.0)	100 (0.1)	99 (0.2)	97 (0.4)	99 (0.3)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

\* Percentage mostly taught before or in the assessment year.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A dash (-) indicates comparable data not available.

An "r" indicates data are available for at least 70% but less than 85% of the students.

**TIMSS Advanced 2015 Algebra Topics**

- 1) **Operations with Expressions:** Operations with exponential, logarithmic, polynomial, rational, and radical expressions
- 2) **Operations with Complex Numbers**
- 3) **Evaluating Algebraic Expressions**
- 4) **Sequences and Series:** The  $n^{th}$  term of arithmetic and geometric sequences and the sums of finite and infinite series
- 5) **Equations and Inequalities:** Linear, simultaneous, and quadratic equations and inequalities; radical expressions, logarithmic, and exponential functions
- 6) **Straight Lines:** Slopes,  $y$ -axis intercepts, and points of intersection of straight lines
- 7) **Representations of Functions:** Equivalent representations of functions, including composite functions, as ordered pairs, tables, graphs, formulas, or words
- 8) **Properties of Functions:** Properties of functions, including domain and range



**Exhibit M9.7: Percentages of Students Taught\* the TIMSS Advanced Topics in Algebra, Calculus, and Geometry (Continued)**

**Calculus Topics**

Reported by Advanced Mathematics Teachers

Country	Calculus Topics						
	Limits of Functions	Continuity and Differentiability	Differentiation	Using Derivatives	Slope and Local Extrema	Points of Inflection	Integrating Functions
France	99 (0.4)	94 (1.4)	98 (0.5)	99 (0.4)	90 (2.0)	17 (2.4)	99 (0.4)
Italy	100 (0.0)	99 (0.3)	100 (0.0)	84 (2.8)	100 (0.3)	99 (0.4)	73 (2.9)
Lebanon	100 (0.1)	99 (0.1)	97 (0.8)	95 (1.1)	99 (0.5)	100 (0.1)	99 (0.1)
Norway	98 (1.5)	97 (2.0)	99 (1.4)	98 (1.6)	98 (1.5)	98 (1.5)	96 (1.7)
Portugal	99 (0.5)	99 (0.7)	96 (1.5)	100 (0.0)	100 (0.1)	100 (0.1)	1 (0.5)
Russian Federation	--	--	--	--	--	--	--
Russian Federation 6hr+	--	--	--	--	--	--	--
Slovenia	98 (1.0)	94 (1.9)	97 (0.9)	82 (3.2)	96 (1.4)	77 (3.1)	74 (1.8)
Sweden	99 (0.6)	97 (1.2)	100 (0.0)	100 (0.4)	100 (0.0)	79 (3.7)	98 (1.1)
United States	r 99 (0.6)	r 98 (0.7)	r 97 (1.4)	r 96 (1.5)	r 96 (1.7)	r 96 (1.7)	r 89 (2.6)
International Avg.	99 (0.3)	97 (0.4)	98 (0.4)	94 (0.6)	97 (0.4)	83 (0.7)	79 (0.6)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

\* Percentage mostly taught before or in the assessment year.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A dash (-) indicates comparable data not available.

An "r" indicates data are available for at least 70% but less than 85% of the students.

**TIMSS Advanced 2015 Calculus Topics**

- 1) **Limits of Functions**
- 2) **Continuity and Differentiability:** Conditions for continuity and differentiability of functions
- 3) **Differentiation:** Differentiation of functions; differentiation of products, quotients, and composite functions
- 4) **Using Derivatives:** Using derivatives to solve problems
- 5) **Slope and Local Extrema:** Using first and second derivatives to determine slope and local extrema of functions
- 6) **Points of Inflection:** Using derivatives to determine points of inflection of functions
- 7) **Integrating Functions:** Integrating functions; evaluating definite integrals, including calculation of areas

### Exhibit M9.7: Percentages of Students Taught\* the TIMSS Advanced Topics in Algebra, Calculus, and Geometry (Continued)

#### Geometry Topics

Reported by Advanced Mathematics Teachers

Country	Geometry Topics			
	Properties of Geometric Figures	Properties of Vectors	Triangles	Trigonometric Functions
France	96 (1.2)	99 (0.4)	98 (0.5)	95 (1.3)
Italy	89 (2.4)	79 (3.2)	99 (0.7)	100 (0.4)
Lebanon	99 (0.1)	100 (0.0)	100 (0.0)	97 (0.7)
Norway	98 (1.0)	100 (0.0)	98 (1.5)	100 (0.0)
Portugal	100 (0.1)	100 (0.0)	100 (0.0)	100 (0.0)
Russian Federation	--	--	--	--
Russian Federation 6hr+	--	--	--	--
Slovenia	100 (0.5)	100 (0.5)	100 (0.5)	100 (0.5)
Sweden	97 (1.1)	93 (1.7)	100 (0.0)	99 (0.6)
United States	r 97 (0.9)	r 70 (4.5)	r 100 (0.1)	r 99 (0.5)
International Avg.	97 (0.4)	92 (0.7)	99 (0.2)	99 (0.2)

SOURCE: IEA's Trends in International Mathematics and Science Study - TIMSS Advanced 2015

\* Percentage mostly taught before or in the assessment year.

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A dash (-) indicates comparable data not available.

An "r" indicates data are available for at least 70% but less than 85% of the students.

#### TIMSS Advanced 2015 Geometry Topics

- 1) **Properties of Geometric Figures:** Properties of geometric figures in two and three dimensions
- 2) **Properties of Vectors:** Properties of vectors and their sums and differences
- 3) **Triangles:** Trigonometric properties of triangles (sine, cosine, and tangent)
- 4) **Trigonometric Functions:** Trigonometric functions and their graphs

**Exhibit M9.8: Percentages of Students Taught the TIMSS Advanced Advanced Mathematics Topics Averaged Across All Topics and by Content Domain\***

Reported by Advanced Mathematics Teachers

Country	All Advanced Mathematics (19 topics)	Algebra (8 topics)	Calculus (7 topics)	Geometry (4 topics)
France	93 (0.4)	98 (0.4)	85 (0.6)	97 (0.6)
Italy	91 (0.6)	89 (0.8)	94 (0.6)	92 (1.2)
Lebanon	98 (0.2)	98 (0.3)	98 (0.3)	99 (0.2)
Norway	92 (0.9)	84 (1.0)	98 (1.3)	99 (0.4)
Portugal	91 (0.3)	93 (0.6)	85 (0.3)	100 (0.0)
Russian Federation	--	--	--	--
Russian Federation 6hr+	--	--	--	--
Slovenia	95 (0.5)	99 (0.3)	88 (1.1)	100 (0.5)
Sweden	94 (0.5)	90 (0.9)	96 (0.6)	97 (0.6)
United States	r 96 (0.5)	r 98 (0.4)	r 96 (1.2)	r 91 (1.2)
International Avg.	94 (0.2)	94 (0.2)	92 (0.3)	97 (0.2)

\* Percentage mostly taught before or in the assessment year.

() Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A dash (-) indicates comparable data not available.

An "r" indicates data are available for at least 70% but less than 85% of the students.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M9.9: National Policies Regarding the Use of Technology in Advanced Mathematics Instruction and Assessment**

Reported by National Research Coordinators

Country	Description of National Policies for Technology Use in Advanced Mathematics Instruction	Description of National Policies for Technology Use in Advanced Mathematics Assessment
France	The policy focuses on using tools such as calculators equipped with Computer Algebra Systems (CAS) in problem solving to focus students on reasoning and strategy rather than technical calculations.	ICT tools are allowed for in-class assessments to assess students' capacity to use technological aids in the process of problem solving. For national examinations, students may use off-line graphing calculators.
Italy	Curriculum guidelines emphasize providing opportunities for students to become familiar with ICT tools and their methodological value; they are not treated as a substitute for all mental calculations.	Same
Lebanon	No policy	No policy
Norway	Digital skills in advanced mathematics involve using digital tools for comprehensive computations and visualization. This means retrieving, processing, and presenting mathematical information in electronic form. It also means evaluating the suitability, possibilities, and limitations of the digital tool.	Every examination in mathematics is now divided into two parts. The first part (3 hours) is solved by pen and paper only; no technological aids are allowed. The second part (2 hours) not only allows the use of some digital tools, but requires that they are applied, such as dynamic geometry programs. It is specifically stated that students in the second part of the exam shall have sophisticated electronic aids available, as long as they cannot use them to communicate.
Portugal	Some subjects (such as normal and binomial distributions) are always taught with graphing calculators.	Some advanced mathematics examinations require the use of a graphing calculator.
Russian Federation	The program has no direct references to the use of electronic devices in advanced mathematics courses. However, the requirements for students' attainment in the subject area "Mathematics and Informatics" include expected learning outcomes for ICT, such as using a computer to construct mathematical models of the proposed situation, conduct experiments, and conduct statistical analysis of data.	No policy. However, during the compulsory state exam in mathematics at Grade 11, students are not allowed to use any calculators or computers. The use of these technological aids in classroom tests depends on the teacher.
Slovenia	Technology is required to be used in teaching and learning. Students are required to demonstrate use of standard and specific software for mathematics. Calculators are not specifically required or described, but teachers and students should use as many devices as possible. In practice, schools require students to have their own calculator capable of symbolic calculations in two lines but not for drawing graphs.	The curriculum does not define the use of calculators for assessments, but on the Matura examination, for all subjects, non-programmable calculators which cannot be connected to the Internet may be used.
Sweden	Digital media and tools are addressed in several curriculum statements as problem solving tools. Mathematics 4 has one additional explicit notion of technology in the description of core content—algebraic and graphical methods for determining integrals with and without digital tools, including estimates of magnitudes and probability distributions.	The grading criteria are very similar for all courses and contain only one statement explicitly referring to technology. Students should be able to solve problems with and without digital tools.
United States	Policies vary by state, but most advanced mathematics courses require graphing calculators and other tools (such as spreadsheets or statistical packages) strategically when solving mathematics problems. Both AP Calculus and IB Mathematics require the use of a graphing calculator to help solve problems, experiment, interpret results, and support conclusions.	Policies vary by state, but some programs (such as AP and IB) have their own specifications about what kinds of calculators are permissible.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M9.10: Availability of Digital Devices in Advanced Mathematics Lessons**

Reported by Advanced Mathematics Teachers

Digital devices may include computers, tablets, calculators, or smartphones.

Country	Digital Devices Available for Students to Use in Advanced Mathematics Lessons		
	Percent of Students	Average Achievement	
	Yes	Yes	No
France	86 (2.2)	460 (3.3)	470 (6.6)
Italy	58 (3.6)	430 (7.4)	417 (10.4)
Lebanon	49 (3.3)	543 (5.2)	521 (3.5)
Norway	100 (0.0)	462 (4.5)	~ ~
Portugal	78 (3.1)	485 (3.0)	477 (5.6)
Russian Federation	64 (4.1)	486 (7.2)	480 (8.0)
Russian Federation 6hr+	68 (4.5)	542 (6.7)	536 (20.6)
Slovenia	75 (2.3)	458 (4.2)	465 (6.4)
Sweden	97 (1.5)	435 (4.2)	445 (8.3)
United States	r 92 (1.6)	485 (6.0)	485 (13.6)
International Avg.	78 (0.9)	472 (1.7)	470 (2.9)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.

An "r" indicates data are available for at least 70% but less than 85% of the students.

**Exhibit M9.11: Profiles of Uses of Digital Devices at Least Monthly in Advanced Mathematics Lessons**

Reported by Advanced Mathematics Teachers

For each country, the percent of students in each use category is plotted along a separate axis. The value of each point is represented as the distance from the center of the graph to illustrate the relative emphasis placed on each use of digital devices in advanced mathematics lessons. Digital devices may include computers, tablets, calculators, or smartphones.



The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

An "r" indicates data are available for at least 70% but less than 85% of the students.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M9.12: Percentages of Students Whose Teachers Have Them Use Digital Devices at Least Monthly in Advanced Mathematics Lessons**

Reported by Advanced Mathematics Teachers

Country	Percent of Students Whose Teachers Have Them Use Digital Devices at Least Monthly							
	Read the Textbook or Course Materials	Look Up Ideas and Information	Process and Analyze Data	Draw Graphs of Functions	Solve Equations	Manipulate Algebraic Expressions	Conduct Modeling and Simulations	Perform Numerical Integration
France	28 (2.9)	48 (3.0)	72 (2.9)	82 (2.4)	72 (2.6)	67 (2.9)	69 (3.2)	64 (3.3)
Italy	31 (2.9)	41 (3.9)	45 (4.0)	44 (3.9)	30 (4.1)	27 (3.7)	31 (3.7)	26 (3.9)
Lebanon	26 (3.8)	35 (3.4)	37 (3.5)	34 (2.9)	45 (3.2)	39 (3.4)	31 (3.0)	42 (3.3)
Norway	48 (6.0)	59 (5.0)	89 (3.5)	99 (1.1)	95 (2.2)	88 (3.0)	92 (3.3)	83 (3.3)
Portugal	30 (4.1)	35 (3.6)	59 (3.9)	77 (3.1)	73 (3.3)	50 (3.3)	72 (3.1)	0 (0.1)
Russian Federation	55 (4.0)	61 (4.1)	50 (3.6)	40 (3.8)	39 (4.0)	38 (3.9)	34 (3.2)	25 (3.8)
Russian Federation 6hr+	60 (5.2)	65 (4.5)	54 (4.2)	44 (4.6)	35 (4.4)	37 (4.4)	38 (4.5)	25 (4.6)
Slovenia	22 (2.5)	41 (3.7)	31 (2.2)	51 (3.7)	39 (3.2)	26 (2.9)	34 (4.5)	17 (2.4)
Sweden	33 (3.3)	61 (3.5)	75 (3.1)	94 (2.0)	87 (2.8)	36 (4.0)	80 (2.8)	89 (2.4)
United States	r 34 (4.2)	r 65 (4.3)	r 75 (3.2)	r 90 (1.8)	r 84 (2.3)	r 71 (3.8)	r 72 (3.3)	r 84 (3.6)
International Avg.	34 (1.3)	50 (1.3)	59 (1.1)	68 (1.0)	63 (1.0)	49 (1.2)	57 (1.1)	48 (1.0)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

An "r" indicates data are available for at least 70% but less than 85% of the students.

**Exhibit M9.13: Profiles of Student Use of the Internet for Advanced Mathematics Schoolwork**

Reported by Advanced Mathematics Students

For each country, the percentage of students in each use category is plotted along a separate axis. The value of each point is represented as the distance from the center of the graph to illustrate the relative emphasis placed on each use of the Internet in advanced mathematics schoolwork.



SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.



**Exhibit M9.14: Percentages of Students Who Use the Internet for Advanced Mathematics Schoolwork**

Reported by Advanced Mathematics Students

Country	Percent of Students Who Use the Internet to Do the Following Tasks						
	Access the Textbook or Other Course Materials	Access Assignments Posted Online by the Teacher	Collaborate with Classmates on Mathematics Assignments or Projects	Communicate with the Teacher	Discuss Mathematics Topics with Other Students	Find Information, Articles, or Tutorials to Aid in Understanding Mathematics Concepts	Find Information, Articles, or Tutorials to Aid in Solving Mathematics Problems
France	50 (1.1)	54 (2.0)	62 (1.1)	29 (1.7)	44 (1.0)	67 (0.9)	74 (0.9)
Italy	50 (1.3)	39 (2.3)	61 (1.1)	37 (1.6)	52 (1.2)	65 (1.4)	63 (1.4)
Lebanon	40 (1.7)	27 (1.9)	63 (1.5)	46 (2.1)	62 (1.7)	46 (1.7)	49 (1.6)
Norway	60 (2.0)	71 (2.6)	47 (1.6)	50 (3.1)	39 (1.7)	74 (1.2)	78 (1.3)
Portugal	41 (1.3)	57 (2.3)	46 (1.4)	31 (1.7)	44 (1.3)	73 (1.1)	75 (1.0)
Russian Federation	78 (0.9)	55 (1.8)	73 (1.0)	22 (1.5)	58 (1.1)	89 (0.5)	86 (0.5)
Russian Federation 6hr+	81 (1.2)	62 (2.6)	76 (1.3)	23 (1.8)	62 (1.3)	88 (0.6)	83 (0.7)
Slovenia	59 (1.3)	55 (2.2)	62 (1.3)	32 (1.8)	52 (1.2)	65 (1.0)	67 (1.1)
Sweden	34 (1.1)	40 (1.7)	34 (1.4)	37 (1.8)	34 (1.3)	59 (1.1)	61 (1.2)
United States	48 (2.2)	54 (2.7)	42 (1.4)	52 (1.9)	33 (1.6)	70 (1.5)	75 (1.6)
International Avg.	51 (0.5)	50 (0.7)	54 (0.4)	37 (0.6)	47 (0.4)	68 (0.4)	70 (0.4)

SOURCE: IEA's Trends in International Mathematics and Science Study - TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

(.) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.



# CHAPTER M10: STUDENT ENGAGEMENT AND ATTITUDES

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS



**IEA**

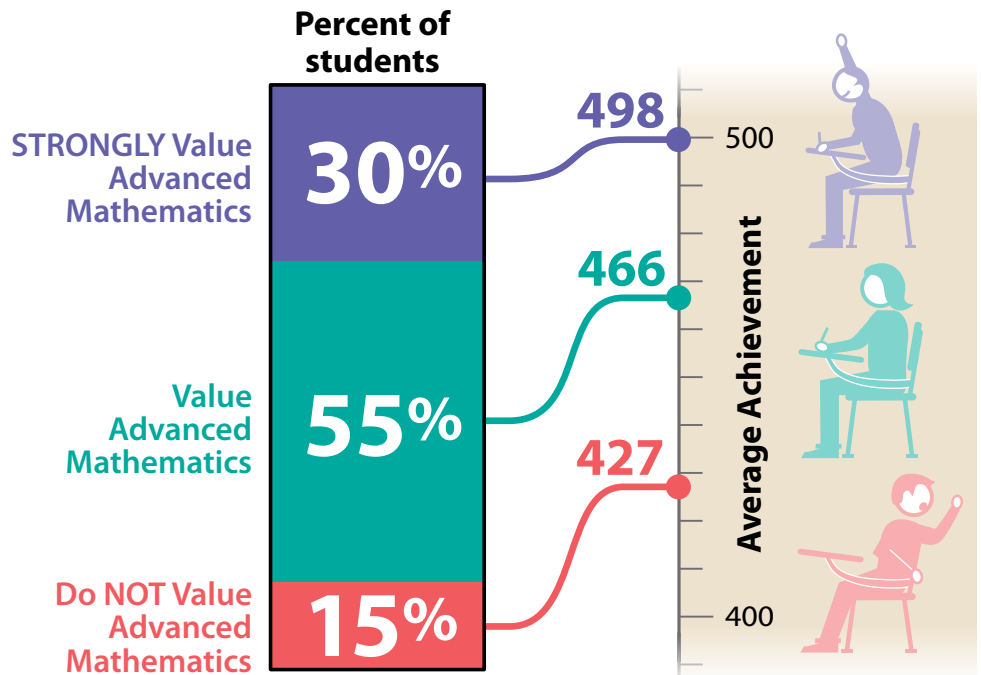
**TIMSS & PIRLS**  
International Study Center  
Lynch School of Education, Boston College



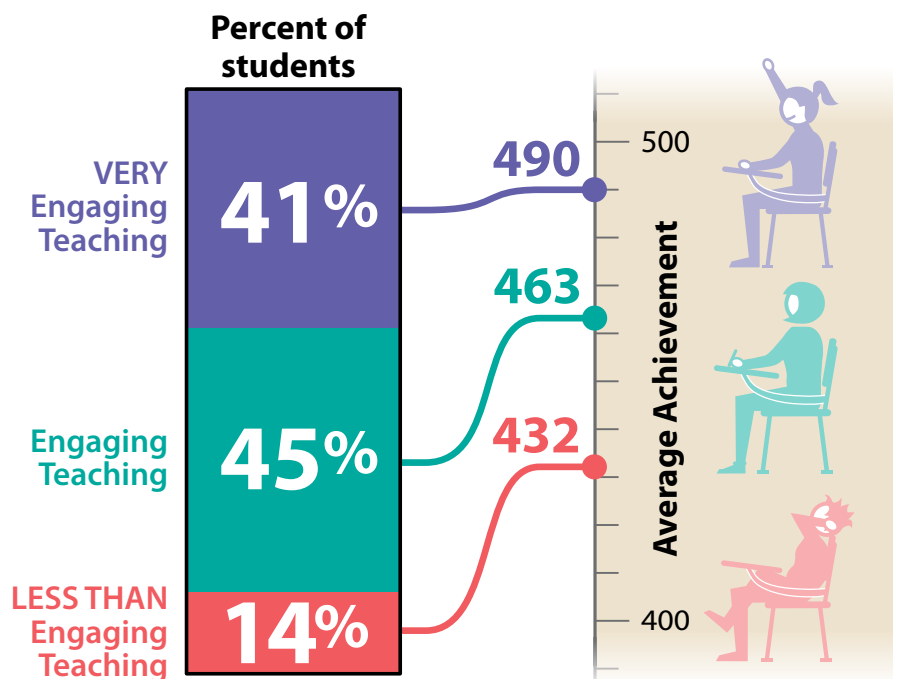
## Students' Attitudes Toward Advanced Mathematics

Most students in advanced mathematics courses had positive attitudes toward mathematics and more positive attitudes were associated with higher achievement.

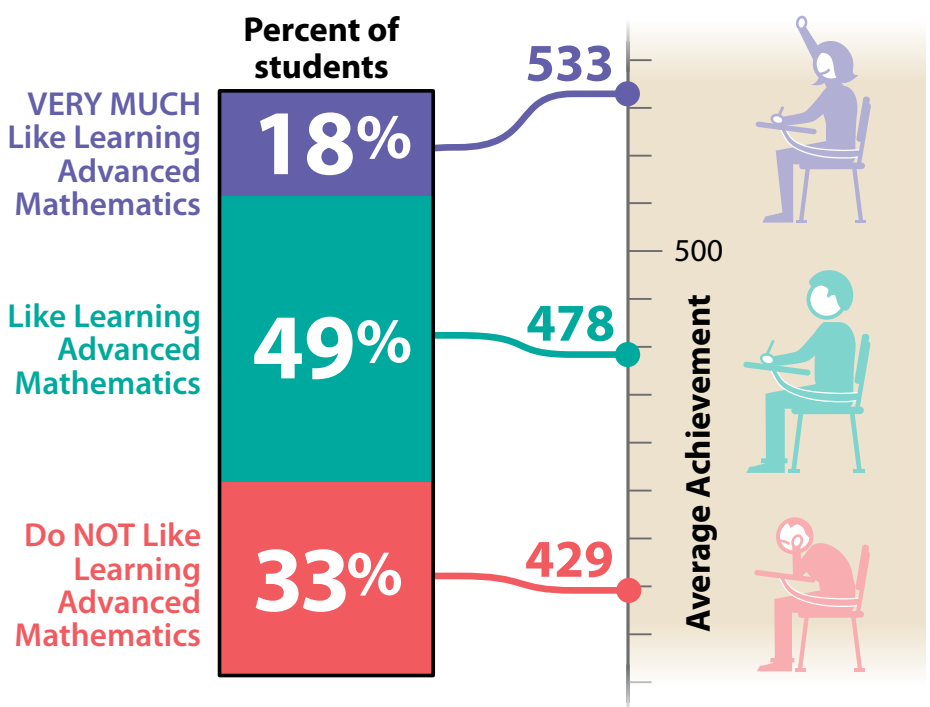
Most students (85%) **STRONGLY** value or value advanced mathematics.



Most students (86%) were positive about their instruction—41% reported **VERY** engaging teaching and 45% engaging teaching.



Students were least positive about actually learning advanced mathematics. The 18% that liked learning it **VERY** much had more than 100 points higher achievement than the 33% that did **NOT** like learning it, again highlighting the strong relationship between excelling at something and liking it.



SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015.  
<http://timss2015.org/advanced/download-center/>





**Exhibit M10.1: Students' Views on Engaging Teaching in Advanced Mathematics Lessons**

Reported by Advanced Mathematics Students

Students were scored according to their degree of agreement with fourteen statements on the *Students' Views on Engaging Teaching in Advanced Mathematics Lessons* scale. Students who experienced **Very Engaging Teaching** in advanced mathematics lessons had a score on the scale of at least 10.4, which corresponds to their "agreeing a lot" with seven of the fourteen statements and "agreeing a little" with the other seven, on average. Students who experienced teaching that was **Less than Engaging Teaching** had a score no higher than 7.9, which corresponds to their "disagreeing a little" with seven of the fourteen statements and "agreeing a little" with the other seven, on average. All other students experienced **Engaging Teaching** in advanced mathematics lessons.

Country	Very Engaging Teaching		Engaging Teaching		Less than Engaging Teaching		Average Scale Score
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	
Lebanon	67 (1.9)	539 (3.3)	28 (1.7)	519 (5.0)	5 (0.6)	525 (13.0)	11.2 (0.08)
Russian Federation 6hr+	60 (2.3)	553 (8.4)	35 (2.0)	523 (8.8)	4 (0.6)	499 (12.4)	10.9 (0.11)
Russian Federation	54 (2.1)	509 (6.2)	38 (1.5)	464 (5.9)	7 (0.9)	413 (12.9)	10.6 (0.09)
United States	54 (1.8)	503 (5.9)	35 (1.4)	473 (8.4)	11 (1.1)	446 (10.5)	10.5 (0.09)
Norway	44 (2.3)	477 (4.9)	47 (1.3)	450 (4.5)	9 (1.2)	422 (6.7)	10.1 (0.09)
Portugal	42 (1.8)	498 (2.9)	44 (1.4)	480 (2.7)	14 (1.3)	446 (5.4)	10.1 (0.09)
France	35 (1.5)	481 (3.5)	57 (1.3)	458 (3.3)	9 (0.8)	421 (5.9)	9.9 (0.06)
Sweden	27 (1.6)	471 (5.0)	51 (1.3)	428 (4.2)	22 (1.6)	391 (6.2)	9.4 (0.09)
Italy	25 (1.4)	429 (8.0)	52 (1.2)	427 (6.3)	24 (1.8)	403 (7.4)	9.2 (0.09)
Slovenia	18 (1.0)	500 (6.4)	57 (1.5)	464 (3.9)	25 (1.6)	425 (3.6)	9.0 (0.05)
International Avg.	41 (0.6)	490 (1.8)	45 (0.5)	463 (1.7)	14 (0.4)	432 (2.9)	

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**How much do you agree with these statements about your advanced mathematics lessons?**

	Agree a lot	Agree a little	Disagree a little	Disagree a lot
1) The teacher clearly communicates the purpose of each mathematics lesson -----	○	○	○	○
2) I know what my teacher expects me to do -----	○	○	○	○
3) My teacher is easy to understand -----	○	○	○	○
4) I am interested in what my teacher says -----	○	○	○	○
5) My teacher gives me interesting things to do -----	○	○	○	○
6) My teacher asks me thought provoking questions -----	○	○	○	○
7) My teacher has clear answers to my questions -----	○	○	○	○
8) My teacher links new content to what I already know -----	○	○	○	○
9) My teacher is good at explaining advanced mathematics -----	○	○	○	○
10) My teacher provides the opportunity for me to show what I have learned -----	○	○	○	○
11) My teacher encourages me to keep working on advanced mathematics problems until I solve them -----	○	○	○	○
12) My teacher provides helpful feedback on my schoolwork (including homework) -----	○	○	○	○
13) My teacher uses a variety of teaching methods, tasks, and activities to help us learn -----	○	○	○	○
14) My teacher believes that I can learn difficult advanced mathematics material -----	○	○	○	○

Very Engaging Teaching 10.4      Engaging Teaching      Less than Engaging Teaching 7.9

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M10.1: Students' Views on Engaging Teaching in Advanced Mathematics Lessons (Continued)**

**Students' Views on Engaging Teaching in Advanced Mathematics Lessons by Gender**

Reported by Advanced Mathematics Students

Country	Very Engaging Teaching		Engaging Teaching		Less than Engaging Teaching	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
<b>Lebanon</b>						
Females	69 (3.4)	539 (5.2)	28 (3.4)	523 (8.5)	4 (0.9)	492 (21.0)
Males	66 (2.0)	538 (4.4)	28 (1.9)	517 (6.2)	5 (0.9)	538 (14.5)
<b>Russian Federation 6hr+</b>						
Females	61 (2.7)	543 (9.9)	35 (2.3)	512 (9.7)	5 (0.9)	482 (17.1)
Males	60 (2.5)	562 (8.1)	36 (2.3)	532 (9.8)	4 (0.6)	515 (13.2)
<b>Russian Federation</b>						
Females	54 (2.9)	503 (7.0)	39 (2.2)	460 (7.0)	7 (1.1)	409 (13.2)
Males	54 (1.9)	514 (6.8)	38 (1.5)	469 (6.5)	8 (0.9)	416 (15.3)
<b>United States</b>						
Females	55 (1.7)	486 (7.1)	34 (1.4)	459 (6.7)	12 (1.2)	428 (12.3)
Males	53 (2.7)	519 (6.0)	37 (2.6)	485 (13.8)	10 (1.3)	465 (13.1)
<b>Norway</b>						
Females	42 (2.9)	471 (5.2)	47 (2.2)	448 (6.0)	11 (1.5)	411 (8.9)
Males	45 (2.4)	480 (5.7)	47 (1.5)	452 (5.4)	8 (1.4)	432 (8.0)
<b>Portugal</b>						
Females	45 (2.2)	496 (3.5)	43 (1.7)	474 (3.8)	12 (1.5)	454 (5.7)
Males	39 (1.9)	499 (4.0)	45 (1.7)	486 (3.2)	16 (1.5)	439 (8.3)
<b>France</b>						
Females	36 (1.9)	465 (3.8)	55 (1.7)	443 (3.5)	9 (1.0)	416 (7.1)
Males	33 (1.7)	496 (4.3)	58 (1.5)	471 (3.8)	8 (1.0)	425 (7.4)
<b>Sweden</b>						
Females	26 (1.9)	466 (5.9)	50 (2.0)	423 (6.0)	24 (2.0)	382 (8.2)
Males	27 (1.9)	474 (6.4)	53 (1.5)	431 (4.2)	20 (1.6)	399 (7.1)
<b>Italy</b>						
Females	25 (1.9)	434 (13.5)	50 (1.8)	431 (6.6)	26 (2.2)	414 (7.9)
Males	25 (1.7)	426 (8.7)	53 (1.3)	425 (7.9)	22 (1.9)	395 (9.2)
<b>Slovenia</b>						
Females	16 (1.1)	485 (6.6)	57 (1.7)	454 (4.3)	27 (2.0)	420 (4.0)
Males	22 (1.5)	516 (8.8)	57 (1.9)	478 (5.2)	21 (1.9)	435 (6.3)
<b>International Avg.</b>						
Females	41 (0.8)	483 (2.3)	44 (0.7)	457 (2.0)	15 (0.5)	425 (3.6)
Males	41 (0.7)	496 (2.1)	46 (0.6)	468 (2.3)	13 (0.5)	438 (3.5)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M10.2: Students Like Learning Advanced Mathematics**

Reported by Advanced Mathematics Students

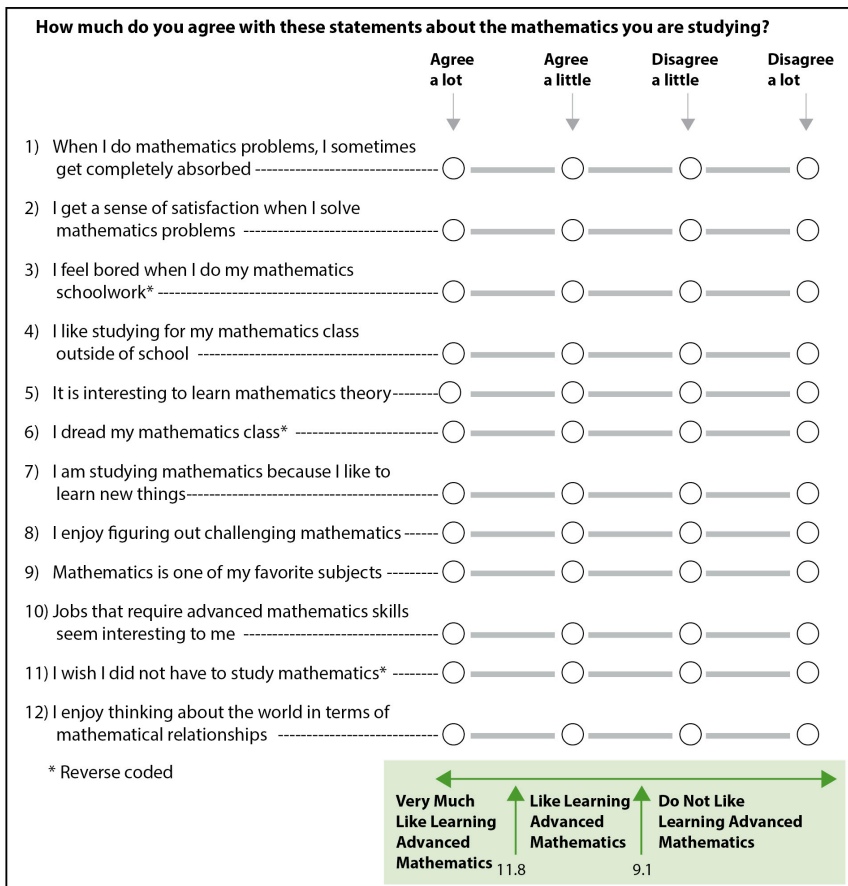
Students were scored according to their degree of agreement with twelve statements on the *Students Like Learning Advanced Mathematics* scale. Students who **Very Much Like Learning Advanced Mathematics** had a score on the scale of at least 11.8, which corresponds to their “agreeing a lot” with six of the twelve statements and “agreeing a little” with the other six, on average. Students who **Do Not Like Learning Advanced Mathematics** had a score no higher than 9.1, which corresponds to their “disagreeing a little” with six of the twelve statements and “agreeing a little” with the other six, on average. All other students **Like Learning Advanced Mathematics**.

Country	Very Much Like Learning Advanced Mathematics		Like Learning Advanced Mathematics		Do Not Like Learning Advanced Mathematics		Average Scale Score
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	
Lebanon	39 (1.8)	553 (4.1)	55 (1.9)	520 (4.4)	6 (0.9)	511 (8.0)	11.5 (0.07)
Russian Federation 6hr+	28 (2.3)	587 (7.9)	49 (1.4)	539 (7.2)	23 (1.6)	484 (11.7)	10.6 (0.11)
Norway	24 (1.2)	509 (5.1)	55 (1.0)	454 (4.5)	21 (1.1)	416 (5.6)	10.5 (0.06)
United States	19 (1.5)	542 (7.2)	50 (1.6)	490 (7.2)	31 (1.4)	445 (6.1)	10.0 (0.07)
Russian Federation	19 (1.2)	549 (6.3)	49 (1.0)	490 (5.9)	32 (1.7)	437 (6.9)	10.1 (0.08)
Portugal	19 (1.0)	537 (3.1)	49 (1.0)	490 (2.7)	33 (1.0)	441 (3.1)	10.0 (0.05)
Sweden	16 (0.8)	518 (4.6)	50 (0.8)	443 (4.2)	35 (0.9)	377 (5.1)	9.9 (0.05)
France	11 (0.6)	529 (4.9)	56 (0.9)	473 (3.0)	33 (1.1)	422 (3.4)	9.8 (0.04)
Italy	9 (0.6)	499 (6.5)	47 (1.0)	441 (5.6)	44 (1.2)	387 (6.8)	9.4 (0.04)
Slovenia	4 (0.5)	559 (8.8)	35 (1.3)	504 (3.3)	61 (1.4)	429 (3.6)	8.7 (0.05)
International Avg.	18 (0.4)	533 (1.9)	49 (0.4)	478 (1.6)	33 (0.4)	429 (1.9)	

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.



SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015



**Exhibit M10.2: Students Like Learning Advanced Mathematics (Continued)**

**Students Like Learning Advanced Mathematics by Gender**

Reported by Advanced Mathematics Students

Country	Very Much Like Learning Advanced Mathematics		Like Learning Advanced Mathematics		Do Not Like Learning Advanced Mathematics	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
<b>Lebanon</b>						
Females	44 (3.0)	547 (7.0)	51 (2.8)	525 (6.5)	5 (1.4)	506 (19.7)
Males	36 (1.9)	557 (4.7)	57 (2.0)	518 (5.1)	7 (1.2)	513 (9.3)
<b>Russian Federation 6hr+</b>						
Females	25 (2.7)	576 (9.5)	49 (1.6)	534 (8.8)	26 (2.6)	477 (14.2)
Males	31 (2.6)	595 (9.0)	49 (2.1)	543 (6.8)	20 (1.4)	492 (10.7)
<b>Norway</b>						
Females	24 (1.9)	497 (5.8)	56 (1.8)	451 (5.8)	20 (1.4)	408 (7.7)
Males	25 (1.6)	516 (5.7)	54 (1.6)	456 (4.6)	21 (1.4)	420 (6.3)
<b>United States</b>						
Females	17 (2.0)	528 (9.2)	45 (2.1)	477 (6.4)	38 (1.9)	437 (6.5)
Males	22 (1.9)	552 (9.1)	54 (1.9)	500 (9.8)	24 (1.6)	457 (8.0)
<b>Russian Federation</b>						
Females	16 (1.0)	544 (6.6)	48 (1.6)	487 (6.6)	36 (2.1)	441 (7.2)
Males	22 (1.6)	553 (7.5)	50 (1.2)	492 (6.3)	28 (1.8)	432 (8.0)
<b>Portugal</b>						
Females	18 (1.3)	531 (3.9)	48 (1.4)	491 (3.4)	34 (1.5)	443 (3.5)
Males	19 (1.3)	543 (4.4)	50 (1.2)	488 (3.3)	31 (1.1)	439 (5.1)
<b>Sweden</b>						
Females	14 (0.8)	506 (7.7)	48 (1.3)	438 (4.6)	38 (1.4)	376 (6.8)
Males	17 (1.1)	525 (4.9)	51 (1.1)	446 (5.1)	33 (1.3)	377 (6.1)
<b>France</b>						
Females	10 (0.7)	509 (5.8)	54 (1.3)	460 (3.3)	36 (1.3)	416 (4.0)
Males	13 (0.8)	543 (5.8)	58 (1.2)	484 (3.3)	30 (1.3)	429 (4.0)
<b>Italy</b>						
Females	9 (1.0)	492 (10.7)	46 (2.0)	450 (7.2)	44 (2.1)	390 (8.0)
Males	9 (0.6)	503 (8.2)	47 (1.0)	435 (6.5)	44 (1.2)	384 (8.5)
<b>Slovenia</b>						
Females	3 (0.5)	545 (12.4)	31 (1.5)	499 (3.9)	66 (1.5)	422 (4.3)
Males	6 (0.9)	570 (9.4)	42 (1.8)	509 (5.6)	53 (2.0)	441 (4.8)
<b>International Avg.</b>						
Females	17 (0.5)	522 (2.7)	47 (0.6)	475 (1.8)	35 (0.6)	427 (2.9)
Males	19 (0.5)	540 (2.3)	51 (0.5)	481 (1.9)	30 (0.5)	433 (2.3)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS-Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

**Exhibit M10.3: Students Value Advanced Mathematics**

Reported by Advanced Mathematics Students

Students were scored according to their degree of agreement with nine statements on the *Students Value Advanced Mathematics* scale. Students who **Strongly Value Advanced Mathematics** had a score on the scale of at least 11.0, which corresponds to their “agreeing a lot” with five of the nine statements and “agreeing a little” with the other four, on average. Students who **Do Not Value Advanced Mathematics** had a score no higher than 8.0, which corresponds to their “disagreeing a little” with five of the nine statements and “agreeing a little” with the other four, on average. All other students **Value Advanced Mathematics**.

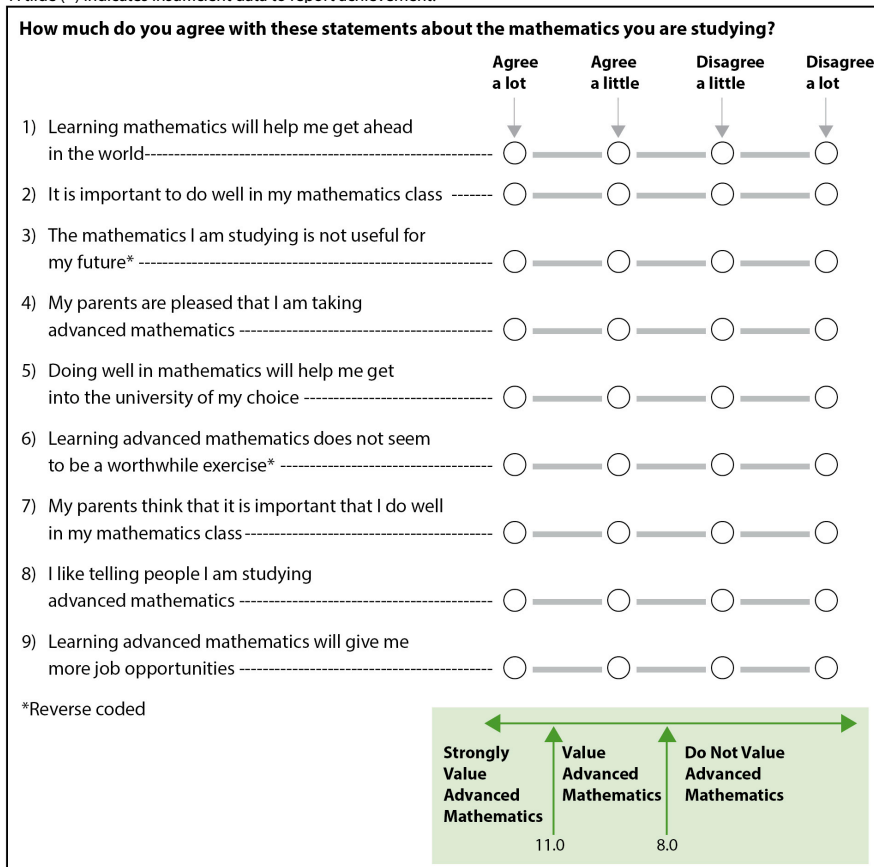
Country	Strongly Value Advanced Mathematics		Value Advanced Mathematics		Do Not Value Advanced Mathematics		Average Scale Score
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement	
United States	54 (1.6)	506 (7.1)	43 (1.3)	463 (5.6)	3 (0.5)	448 (13.6)	11.3 (0.07)
Lebanon	47 (1.5)	547 (4.2)	50 (1.5)	522 (3.8)	3 (0.7)	492 (13.3)	11.1 (0.06)
Portugal	41 (1.4)	509 (2.8)	51 (1.2)	469 (3.1)	8 (0.6)	432 (4.9)	10.5 (0.05)
Norway	40 (1.2)	475 (5.1)	56 (1.2)	452 (5.0)	5 (0.6)	418 (7.3)	10.6 (0.05)
Russian Federation 6hr+	36 (2.2)	567 (7.1)	51 (1.4)	537 (7.2)	12 (1.4)	473 (17.5)	10.3 (0.12)
Russian Federation	26 (1.4)	525 (6.3)	56 (0.6)	482 (5.7)	18 (1.1)	433 (7.5)	9.8 (0.08)
Sweden	26 (0.9)	461 (5.1)	64 (1.0)	426 (4.5)	10 (0.6)	391 (6.9)	10.0 (0.04)
Italy	18 (0.9)	457 (7.1)	59 (1.0)	428 (5.9)	24 (1.0)	383 (7.7)	9.3 (0.05)
France	15 (0.7)	503 (4.8)	69 (0.8)	464 (2.8)	16 (0.8)	419 (4.1)	9.4 (0.04)
Slovenia	2 (0.3)	~ ~	50 (1.6)	486 (4.0)	48 (1.6)	430 (3.7)	8.2 (0.04)
International Avg.	30 (0.4)	498 (1.9)	55 (0.4)	466 (1.5)	15 (0.3)	427 (2.8)	

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

A tilde (~) indicates insufficient data to report achievement.



SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Exhibit M10.3: Students Value Advanced Mathematics (Continued)**

**Students Value Advanced Mathematics by Gender**

Reported by Advanced Mathematics Students

Country	Strongly Value Advanced Mathematics		Value Advanced Mathematics		Do Not Value Advanced Mathematics	
	Percent of Students	Average Achievement	Percent of Students	Average Achievement	Percent of Students	Average Achievement
<b>United States</b>						
Females	52 (1.8)	492 (6.7)	45 (1.7)	447 (5.3)	3 (0.6)	441 (20.3)
Males	55 (2.4)	519 (9.5)	41 (1.9)	481 (8.1)	4 (1.0)	454 (17.4)
<b>Lebanon</b>						
Females	52 (2.6)	543 (5.8)	46 (2.5)	525 (6.5)	2 (0.6)	~ ~
Males	44 (2.1)	550 (5.3)	53 (2.3)	521 (4.8)	3 (0.9)	487 (13.4)
<b>Portugal</b>						
Females	38 (1.6)	508 (3.6)	54 (1.5)	469 (3.6)	8 (0.6)	437 (5.2)
Males	43 (1.8)	510 (3.7)	49 (1.6)	469 (4.4)	8 (0.9)	427 (7.6)
<b>Norway</b>						
Females	36 (1.8)	465 (5.7)	59 (1.8)	450 (6.6)	5 (0.8)	409 (12.2)
Males	42 (1.3)	480 (6.4)	53 (1.4)	453 (5.2)	5 (0.7)	423 (7.6)
<b>Russian Federation 6hr+</b>						
Females	33 (2.6)	560 (8.4)	53 (1.5)	527 (8.0)	14 (2.2)	465 (22.5)
Males	39 (2.2)	572 (7.6)	50 (1.7)	546 (7.4)	11 (1.0)	481 (15.2)
<b>Russian Federation</b>						
Females	23 (1.3)	520 (7.5)	57 (0.9)	480 (6.2)	20 (1.3)	435 (8.3)
Males	30 (1.6)	529 (7.0)	54 (0.9)	485 (6.2)	16 (1.2)	430 (8.6)
<b>Sweden</b>						
Females	25 (1.2)	448 (6.5)	65 (1.3)	420 (5.3)	10 (0.8)	392 (10.8)
Males	27 (1.3)	469 (6.0)	64 (1.4)	430 (5.5)	9 (0.8)	390 (9.5)
<b>Italy</b>						
Females	16 (1.4)	461 (12.6)	61 (1.5)	432 (6.7)	24 (1.6)	394 (11.4)
Males	19 (1.0)	456 (8.1)	57 (1.6)	425 (7.8)	24 (1.3)	376 (8.5)
<b>France</b>						
Females	12 (0.8)	486 (5.1)	70 (1.2)	452 (3.3)	18 (1.0)	416 (5.0)
Males	17 (0.9)	513 (5.8)	68 (1.1)	476 (3.2)	14 (1.1)	424 (4.9)
<b>Slovenia</b>						
Females	2 (0.4)	~ ~	44 (1.6)	476 (3.8)	54 (1.6)	425 (4.6)
Males	3 (0.5)	557 (12.1)	58 (2.4)	497 (5.4)	40 (2.5)	442 (5.4)
<b>International Avg.</b>						
Females	28 (0.5)	490 (2.5)	56 (0.5)	461 (1.8)	16 (0.4)	418 (3.8)
Males	31 (0.5)	509 (2.5)	55 (0.6)	471 (1.9)	14 (0.4)	428 (3.3)

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS-Advanced 2015

The Russian Federation 6hr+ results are for a subset of the Russian Federation students. This subset of students are in an Intensive stream that have at least 6 hours of mathematics lessons per week.

This TIMSS Advanced questionnaire scale was established in 2015 based on the combined response distribution of all countries that participated in TIMSS Advanced 2015. To provide a point of reference for country comparisons, the scale centerpoint of 10 was located at the mean of the combined distribution. The units of the scale were chosen so that 2 scale score points corresponded to the standard deviation of the distribution.

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**CHAPTER M11:  
DESCRIPTION OF  
ADVANCED MATHEMATICS  
PROGRAMS AND  
CURRICULUM**

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS



**IEA**

**TIMSS & PIRLS**  
International Study Center  
Lynch School of Education, Boston College



## Description of the Advanced Mathematics Programs and Curriculum

### France

The Grade 11 and 12 scientific track offers robust mathematical knowledge and skills to students aiming for careers in science, technology, engineering, and mathematics (STEM). The mathematics curriculum is meant to develop students' scientific thinking and strengthen their interest in and affinity for scientific research. Together with introducing new mathematical knowledge and content, the curriculum targets developing students' skills and mathematical faculties in these areas:

- ◆ Implementing mathematical investigations and employing a variety of problem solving strategies
- ◆ Mastering a wide range of reasoning processes
- ◆ Interpreting and validating mathematical results
- ◆ Communicating mathematics both orally and in writing

Mathematical activities assigned to students both in class and for homework are focused on intra-mathematical or contextually diverse problem solving situations. Students are trained in:

- ◆ Searching for information, experimenting, and modeling, all using technology
- ◆ Choosing and executing calculation techniques
- ◆ Implementing algorithms
- ◆ Reasoning, proving, and validating results
- ◆ Explaining an answer, communicating a result

The mathematical content is organized in three parts: Analysis, Geometry, Probability and Statistics. About half of class time should be devoted to Analysis, one quarter to Geometry, and the last quarter to Probability and Statistics. The topics included in each content area are listed below.

Content Area	Topics
Analysis	<p>Quadratic functions: solving quadratic equations, sign of a quadratic function</p> <p>Sequences: arithmetic and geometric sequences, induction, finite or infinite limits, bounded sequences</p> <p>Function limits: finite or infinite limits, limits of a sum, product, quotient or composite functions, asymptotes</p> <p>Continuity on an interval, including the Intermediate Value Theorem</p> <p>Differentiation: calculating derivatives, including the derivatives of common functions, derivatives of sums, products, and quotients of functions, and applications of derivatives, including the relationship between the intervals over which a function increases or decreases and the value of its derivative on those intervals and function extrema</p>

Content Area	Topics
Analysis (Continued)	<p>Sine and cosine functions</p> <p>Exponential functions</p> <p>Natural logarithms</p> <p>Integration on an interval, including the relationship between the definite integral and the area under a curve, notation, the antiderivative of a function, linearity, and the additive property of definite integrals</p>
Geometry	<p>Complex numbers, including the algebraic form, conjugate, geometric representation, and polar form of a complex number; the sum, product, and quotient of complex numbers, complex solutions to quadratic equations</p> <p>Euclidean vectors, including the characterization of a line and a plane, scalar product, coordinates, equation of a plane</p> <p>Trigonometry, including trigonometric functions defined on the unit circle, radian, the sine and cosine of supplementary and complementary angles</p>
Probability and Statistics	<p>Descriptive statistics, including variance, standard deviation</p> <p>Conditional probability, independence</p> <p>Probability density functions, including discrete and continuous random variables, probability distributions (normal, Bernoulli, binomial, uniform, exponential), variance, standard deviation</p> <p>Confidence intervals</p> <p>Sampling, confidence interval for a proportion</p>

Starting in Grade 10, scientific track students continue to develop and implement algorithms. Students are trained to:

- ◆ Describe algorithms in natural or symbolic language
- ◆ Devise basic algorithms using spreadsheets, calculators or specific software programs
- ◆ Interpret complex algorithms

Algorithms fit naturally in all mathematical fields. Algorithmic problem solving in each content area is situated in contexts related to academic subject areas and contexts from real life. Students learn how to implement elementary instructions, loops, and conditional instructions as well as to implement validation and control steps in their programs.

Students learn how to use formal mathematical notation (e.g., for functions, derivatives, and integrals) as well as notation for number sets and intervals.

Students learn elements of formal logic, such as the logical operators for “and” and “or”; the concepts of the contrapositive, the converse and the negative of a conditional statement; logical equivalence; types of arguments, such as the counterexample, the logical disjunction, and the contrapositive; and proof by contradiction.

## Italy

Students assessed in TIMSS Advanced 2015 have been taught according to the 2010 National Guidelines for upper-secondary schools (*Licei, Istituti Tecnici, Istituti Professionali*). Only students of *Liceo Scientifico* (high schools specializing in science education) and of *Istituto Tecnico–Settore Tecnologico* (technical high school–technology sector) participated in the TIMSS Advanced 2015 advanced mathematics assessment. In fact, these are the only tracks with elements of advanced mathematics in their curricula, with an appropriate lesson time (4 hours per week, for 33 weeks in a school year).

The mathematics curriculum for upper-secondary school includes four main areas:

- ◆ Arithmetic and Algebra
- ◆ Geometry
- ◆ Relations and Functions
- ◆ Statistics and Probability

*Liceo Scientifico* focus on the study of the link between the scientific and the humanistic traditions. They promote the acquisition of knowledge and methods of mathematics, physics, and the natural sciences. At the conclusion of the program of study, students should be able to:

- ◆ Understand the formal language of mathematics, know how to use typical mathematical procedures, and know the basic content of the theories underling the mathematical description of reality
- ◆ Use data processing tools critically and in-depth; understand the methodological value of information technology in formalizing and modelling complex processes; and identify procedures that lead to conclusions and judgments about real-world systems modelled by data
- ◆ Understand the fundamental structures of mathematical argumentation and demonstrate mathematical processes through the mastery of the language of formal logic; use mathematical argumentation and formal logic to identify and to solve problems of various kinds
- ◆ Know how to use computational and representation tools for modelling and solving problems

At the end of the course, students of *Liceo Scientifico* must know the basic concepts and methods of mathematics and apply them to describe and predict phenomena in the physical world. They can situate mathematical theories in historical context and understand their conceptual meaning. The *Liceo Scientifico* five-year curriculum is divided into three parts by grade. The topics taught at each grade are listed below.



Grade	Topics
Grades 9 and 10	<p>Arithmetic: integer, rational and real numbers; algebra, polynomials, algebraic equations of first and second-degree, inequalities, simultaneous equations</p> <p>Functions: Linear functions <math>f(x) = ax + b</math>, quadratic functions <math>f(x) = ax^2 + bx + c</math>, <math>f(x) =  x </math>, and <math>f(x) = a/x</math></p> <p>Euclidean geometry and Cartesian plane geometry: geometric transformations in the plane, circles, circumference, and <math>\pi</math> (pi), introduction to trigonometric functions and to vectors</p> <p>Descriptive statistics: average values, variance, standard deviation</p> <p>Classic probability, probability theorems</p>
Grades 11 and 12	<p>Analytic geometry</p> <p>Conics</p> <p>Spatial geometry: planes, lines, polyhedra, pyramids, solids of rotation (cylinder, cone, sphere), areas and volumes of elementary solids</p> <p>Trigonometry: triangles, law of sines and law of cosines law</p> <p>Trigonometric functions: trigonometric equations and inequalities</p> <p>Exponential functions: exponential equations and inequalities</p> <p>Logarithmic functions, <math>e</math> (base of natural logarithms), logarithmic equations and inequalities</p> <p>Arithmetic and geometric sequences and series</p> <p>Mathematical induction</p> <p>Complex numbers: algebraic, geometric and trigonometric forms and representations; sums, products, and quotients; complex solutions of quadratic equations</p> <p>Combinatorics</p> <p>Statistics: regression and correlation</p> <p>Conditional probability, Bayes' theorem</p>
Grade 13	<p>Limit of a series</p> <p>Functions</p> <p>Limits: finite or infinite limits, limits of sums, products, quotients or composite functions, asymptotes</p> <p>Continuity on an interval</p> <p>Differentiation: numerical derivatives; the derivatives of common functions; derivatives of sums, products, quotients, and composite functions; applications of derivatives; the relationship between differentiability and continuity; the fundamental theorems of differential calculus; maxima and minima</p> <p>Integration: integration on an interval, the relationship between the definite integral and the area under a curve, the antiderivative of a function, applications of definite integrals</p> <p>Differential equations and applications, particularly in physics</p> <p>Analytic spatial geometry: coordinates, equations of planes, lines, spheres</p> <p>Probability: discrete and continuous random variables, probability distributions (Bernoulli, Poisson, normal), variance, standard deviation</p>

The topics taught at each grade at *Istituti Tecnici–Settore Tecnologico* are essentially the same as those listed above, but have a more applicative orientation at Grades 11, 12 and 13. Also it should be noted that some topics of calculus (function limits, continuity, derivatives) are taught in

Grade 12, instead of Grade 13 as in *Liceo Scientifico*, while the integral calculus is taught in Grade 13 as in *Liceo Scientifico*. In these technical institutes, at Grades 12 and 13, many mathematical topics that serve specific technological applications are taught, such as partial derivatives, Fourier series, Taylor's formula, spherical trigonometry, etc.

In this type of high school, at the end of the five-year course, the study of mathematics helps students achieve the following learning outcomes:

- ◆ Mastery of formal language and demonstration procedures of mathematics
- ◆ Possession of the mathematical, statistical, and probability tools necessary for the understanding of scientific disciplines and the ability to work in the field of applied science
- ◆ Understanding of the place of mathematics in the history of science

## Lebanon

At Grade 12, students receive a solid mathematical foundation with the aim of preparing them to pursue their studies as teachers, engineers, and researchers. The mathematics competencies students must have in each domain are provided in the table below.

Domain	Competencies
Mathematical Reasoning	<ul style="list-style-type: none"> <li>Recognize the difference between a mathematical explanation and concrete or experimental evidence</li> <li>Make conjectures and discover means to test them</li> <li>Carry out proofs using various modes of reasoning</li> <li>Analyze and prove a statement of necessary and sufficient conditions</li> <li>Recognize a universal statement, a statement of existence, and a statement of uniqueness</li> <li>Evaluate a mathematical argument and criticize a proof</li> <li>Carry out an inductive proof</li> </ul>
Problem Solving	<ul style="list-style-type: none"> <li>Formulate a problem out of situations studied in mathematics, in other sciences, or encountered in real life</li> <li>Use various mathematical interpretations to represent the information given in a problem, figure out a convenient strategy to solve it, and take various approaches to make this strategy work using mathematical knowledge</li> <li>Discuss the validity of obtained solutions</li> </ul>
Communication	<ul style="list-style-type: none"> <li>Give a summary of a mathematical document</li> <li>Take notes on a mathematical lecture</li> <li>Critique a mathematical presentation</li> <li>Write a proof correctly</li> </ul>
Spatial	<ul style="list-style-type: none"> <li>Prove and apply the properties of solid figures and conics</li> <li>Characterize plane or space figures using vector notation</li> <li>Study geometric problems analytically</li> <li>Determine the effect of transformations on plane figures</li> </ul>
Numbers and Algebra	<ul style="list-style-type: none"> <li>Analyze the extensions of the sets of numbers (natural numbers, integers, rational numbers, real numbers, complex numbers)</li> <li>Study the properties of complex numbers and their use in geometry and trigonometry</li> <li>Generalize fundamental mathematical notions (set, relation, binary operation, and propositional calculus)</li> <li>Acquire an example of structure</li> <li>Develop mathematical tools for numerical calculations, and for solutions of systems of equations and inequalities</li> </ul>
Calculus	<ul style="list-style-type: none"> <li>Acquire the fundamental concepts of limit, continuity, and differentiability, and use them graphically</li> <li>Analyze the graphs of polynomial, rational, irrational, trigonometric, logarithmic, and exponential functions</li> <li>Integrate functions and solve simple differential equations</li> </ul>
Statistics and Probability	<ul style="list-style-type: none"> <li>Organize information and represent it graphically</li> <li>Study the characteristics of a statistical distribution of one variable</li> <li>Solve simple probability problems, mainly in discrete cases where the events are equally likely</li> </ul>

## Norway

TIMSS 2003 and PISA 2003 showed a decrease in Norwegian students' performance in mathematics and science in compulsory school compared with TIMSS 1995 and PISA 2000. This resulted in a broad discussion about how to improve the learning outcomes in Norway. A big effort was made to change the curriculum for all subjects in all 13 grades. There was an agreement nationally that something had to be done, and the new curriculum received support across all political parties in the parliament. It was called the Knowledge Promotion Reform, and was implemented in the autumn of 2006. The last cohort using the previous curriculum was in Grade 13 in the 2007–2008 school year, which means that these students were assessed in TIMSS Advanced 2008. Students assessed in TIMSS Advanced 2015 have been taught according to the 2006 curriculum.

In the present curriculum, two features stand out. First, the learning goals are formulated as competencies. Second, there are five basic skills (literacies) which are supposed to be used and developed in all subjects and at all levels: the ability to express oneself orally, the ability to read, the ability to express oneself in writing, the ability to use digital tools, and numeracy. Digital devices are supposed to be widely used in teaching, learning, and testing.

The following table indicates topics taught in the courses Mathematics R1 and Mathematics R2, normally taken in Grades 12 and 13, respectively.

Content Area	Topics
Geometry (R1 and R2)	Selected elements of Euclidean plane geometry, including geometric loci and similarity; constructions with compass and straightedge, and with geometry software; the intersection theorems for heights, angle bisectors, perpendicular bisectors and medians in a triangle; various proofs of Pythagoras' Theorem; vectors in the plane, with and without coordinates; application of vectors to determine lengths, angles, and parallelism and orthogonality of lines; vectors in space, with and without coordinates; application of scalar and vector products to determine distances, angles, areas and volumes; representation of lines, planes, and spheres by equations and in parametric form; calculation of lengths, angles and areas in bodies limited by planes and spheres
Algebra (R1 and R2)	Division and factorization of polynomials; logarithms; polynomial, rational, and logarithmic equations and inequalities; transformation and simplification of rational functions and other symbolic expressions with and without the use of digital aids; direct and contrapositive proof; proof by induction; number patterns; finite arithmetic series; finite and infinite geometric series; convergence
Functions (R1 and R2)	Limit, continuity and differentiability; derivatives of polynomial, exponential, and logarithmic functions; derivatives of sums, differences, products, and quotients of functions, and of composite functions; interpretation of functional behavior from the first and second derivatives; interpretation of derivatives in models of practical situations; drawing function graphs by hand and by digital tools; interpretation of a function's basic properties from its graph; horizontal and vertical asymptotes; vector functions with parameters; velocity and acceleration as derivatives of vector functions; trigonometric functions and equations; derivatives of trigonometric functions;

Content Area	Topics
Functions (R1 and R2) (Continued)	modeling periodic phenomena; definite and indefinite integrals; integration by substitution, by parts, and by partial fractions with linear denominators; interpretations of definite integrals in practical applications, and calculation of areas of plane regions and volumes of solids of revolution; mathematical modeling based on observed data
Combinatorics and Probability (R1 only)	Independence and conditional probability; Bayes' theorem for two events; ordered samples with and without replacement; unordered samples without replacement; applications to calculation of probabilities
Differential Equations (R2 only)	Modeling practical situations by differential equations; interpretation of solutions; linear first order and separable differential equations; second order homogenous differential equations; the use of Newton's second law and second order differential equations to describe free oscillations by periodic functions; application of digital tools to draw vector diagrams and integral curves

The previous curriculum for advanced mathematics covered quite a bit of statistics, including binomial, hypergeometric, and normal distributions, confidence intervals, and hypothesis testing. This was an important part of the curriculum in both of the advanced mathematics courses. The present curriculum has much less on statistics. The remaining parts are some combinatorics and probability taught in the first year of the advanced mathematics track (Mathematics R1). Another important change in the curriculum is that mathematical proof is emphasized more in the present curriculum than in the previous one. The new curriculum states that students shall “give an account of implication and equivalence, and implement direct and contrapositive proof” the first year (Mathematics R1) and “implement and give an account of proof by induction” the second year (Mathematics R2).

There have only been minor adjustments made to the curriculum after 2006. Both the new and the previous curricula emphasize the use of digital tools in mathematics. Under previous curricula, a liberal policy was developed to encourage and allow an extensive use of aids in all teaching and testing. Written notes and advanced calculators were normally allowed in local tests as well as in national written examinations. This has changed in the present curriculum. Every exam in mathematics is now divided into two parts. The first part is solved by pen and paper only and no aids are allowed. The second part, however, does not only allow the use of digital tools, but some are even required, like dynamic geometry programs. It is specifically stated that students in the second part of the exam shall have quite sophisticated electronical aids available.

Not all students have to take a national written exam in mathematics. About 40 percent of the first year (Mathematics R1) students are sampled, as are about 60 percent the second year (Mathematics R2) students. For the local oral exam, about 5 percent and 15 percent of the students in the respective courses are sampled for testing.

There is no national certification of teaching materials, such as textbooks, in Norway. The authors and publishers are free to decide the content of a textbook; the responsibility for covering the national curriculum rests on the school and the teacher.

Generally, one may say that the present curriculum emphasizes pure mathematics a little more than the previous one, across all levels. For instance, the present curriculum has a slightly stronger emphasis on algebra in compulsory school. Also, as has already been mentioned, formal proofs are now more emphasized than before in the advanced mathematics courses of upper-secondary school.

## Portugal

Advanced Mathematics is a mandatory course for students in the upper-secondary Science and Technology and Socioeconomic Sciences academic tracks. The curriculum is divided into three main subjects: Probability and Combinatorics, Introduction to Differential Calculus II, and Trigonometry and Complex Numbers. The topics included in each main subject are listed below.

Main Subject	Topics
Probability and Combinatorics	<p>Introduction to probability: random experiments; outcome spaces; events and operations with events; classical, frequency and axiomatic definitions of probability; conditional probability and independence of events</p> <p>Relative frequency and probability distributions: random variables and density functions for discrete variables; sample versus population means and standard-deviations; binomial probability distributions; normal distributions; histograms versus continuous probability density functions</p> <p>Combinatorics: enumerative combinatorics; permutations and combinations; Pascal's Triangle and Newton's Binomial expansion; the Binomial Theorem; applications of probability calculations</p>
Introduction to Differential Calculus II	<p>Exponential and logarithmic functions: analytical and graphical properties of exponential and logarithmic functions; rules for exponents and logarithms; modeling with exponential and logarithmic functions</p> <p>Limits theory: Heine's definition of the limit of a function and its properties; notable special limits; indeterminate forms of limits; asymptotes; continuity of functions, Bolzano-Cauchy's Theorem; numerical applications</p> <p>Differential calculus: Derivatives rules and applications; concavity and second derivatives; composite functions and their derivatives; properties of simple functions that can be determined by studying derivatives; optimization problems</p>
Trigonometry and Complex Numbers	<p>Trigonometry: intuitive study of the sine, cosine and tangent functions and their derivatives based on the unit circle; special limits of the sine function; use of trigonometry functions in modeling</p> <p>Complex numbers: introduction to complex numbers; the imaginary unit; algebraic form of and operations with complex numbers in this form; trigonometric form of complex numbers and operations with complex numbers in this form; geometric interpretation of operations with complex numbers; complex variables in the geometric plane</p>

## Russian Federation

High school programs for mathematics (Grades 10-11) are distinguished by the amount of the material being studied and the amount of instructional time. The Basic level program is designed for those students who plan to learn a profession that is not related to mathematics or plan to use mathematics as an auxiliary “tool” in their professional lives. The Profile level program provides sufficient depth of mathematics study to make it possible for students to enter a profession where mathematics is actively used. It includes a large amount of content and has higher requirements for its mastery. The mastery of this content makes it possible for students to continue to university-level studies in mathematical disciplines. Within the Profile level there is a subset of students in an even more intensive program taking six hours or more of mathematics lessons per week. The sample of students participating in the TIMSS Advanced 2015 Advanced Mathematics assessment included both Profile-level students and Intensive-level students. The results for students in the Intensive level were also reported separately as Russian Federation 6hr+.

The Profile level curriculum includes an explanation of the main goals of the program and provide for the organization and planning of mathematics courses, including:

- ◆ General characteristics of the profile course
- ◆ Teaching goals
- ◆ The number of lessons per week and per year
- ◆ General learning skills and activities
- ◆ Compulsory content and learning outcomes

The content of the Profile course is divided into two sections: Algebra and Calculus, and Geometry. The topics included in each section are listed below.

### Content Areas in Algebra and Calculus

#### Grade 10

Polynomials	Transformation of polynomials, factorization; division of polynomials; Horner’s method; roots of polynomials; Bezout’s theorem; converting irrational expressions
Graphs of Functions	Complex functions; conversion of graphs; graphs of linear-fractional functions, asymptotes; graphs of functions which include a sign of a module (e.g., $y = \frac{2x-6}{ 3-x }$ or $y = \sin x $ ); reciprocal functions and their graphs
Introduction to Calculus	Numerical sequences, limits of sequences, limits of functions, theorems on limits of functions; properties and continuity of elementary functions
Derivatives and their Applications	Geometric and physical meaning of the derivative, continuity and differentiability of functions, derivatives of sums, products, quotients, composites and exponential functions; second derivatives and higher order derivatives; application of derivatives to study functions; Lagrange’s theorem and its consequences; drawing graphs of functions



## Content Areas in Algebra and Calculus

Trigonometric Functions	Trigonometric functions of numeric argument (sine, cosine, tangent and cotangent); trigonometric identities and their consequences; reduction formulas; identical transformation of trigonometric expressions; periodicity of trigonometric functions; properties, graphs, and derivatives of trigonometric functions
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### Grade 11

Integral and Differential Equations	Indefinite integrals; definite integrals and their properties, numerical approximation of definite integrals, approximate computation; Newton-Leibniz formula; application of integrals for calculating areas, volumes, and lengths of arcs in physical problems; solutions of simple differential equations
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Exponential and Logarithmic Functions	Properties and graphs of exponential functions; logarithms, definitions, and properties; identical transformations of exponential and logarithmic expressions; exponential and logarithmic equations, inequalities and systems of inequalities, types and methods of solution; derivatives of exponential functions; natural logarithms, radioactive decay
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Complex Numbers	Algebraic form, arithmetic operations, conjugating complex numbers; solutions of quadratic equations with complex coefficients; the complex plane; trigonometric form of complex numbers, multiplication, division, and raising to power; De Moivre's formula; complex roots of polynomials; the Fundamental Theorem of Algebra
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Elements of Combinatorics	Methods of mathematical induction; proofs of identities; factorials; the basic formulae of combinatorics; combinations and permutations; Binomial Theorem, Dirichlet's Principle
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Elements of the Theory of Probability and Mathematical Statistics	Classic definition of probability, calculating probabilities using combinatorics; conditional probability, the rules of addition and multiplication of probabilities, independent events, Bernoulli distribution; mathematical expectation and variance; the concept of the law of large numbers and a normal distribution law; parent population and sample, levels of significance and reliability; evaluation of probability using frequency; the concept of statistical hypothesis testing
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Equations, Inequalities, Systems	General methods and approaches for solving equations; irrational equations; generalized method of intervals for solving inequalities; systems of equations and inequalities, basic methods for solving systems of equations; application of graphs to solve equations, inequalities and systems; approximate methods for solving equations; equations, inequalities, and systems with parameters
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## Content Areas in Geometry

### Grade 10

#### Axioms of Solid Geometry

Parallel Lines and Planes	Mutual arrangement of lines and planes in space; theorems of parallelism of lines and planes
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Perpendicularity of Lines and Planes	Theorems of dependences between parallelism and perpendicularity of lines and planes, the Theorem of the Three Perpendiculars; angles between straight lines and a plane
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Coordinates and Vectors in a Space	Rectangular coordinate systems on a plane, the formula for distance between points, equations of straight lines and circumference; Cartesian coordinate system in a space, equations of straight lines and a plane; movements in a space and their properties (central symmetry, parallel translation, rotation), similarity in a space
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Vectors in a Space	Decomposition of vectors into three non-coplanar vectors; scalar products; applications of coordinates and vectors to solve problems
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**Content Areas in Geometry**

**Grade 11**

Polyhedrons	Concepts of polyhedrons, prisms, rectangular parallelepipeds, and pyramids; areas of faces and surfaces; sections; regular polyhedrons; dihedral angles
Solids of Revolution	Bodies and surfaces of revolution, cylinders, cones, axial sections of cylinders and cones; spheres and solid spheres, sections of solid spheres, equation of a sphere; inscribed and circumscribed cylinder, cone, sphere
Volumes of Bodies	Volumes of polyhedrons (prisms, pyramids) and solids of revolution (cylinder, cone, sphere, part of the sphere)
The Surface Areas of Solids of Revolution	Areas of spheres, surface areas of cylinders and cones

Learning outcomes are described in terms of what students should know and be able to do in each of these areas. Teachers have some discretion as to the introduction of optional topics.

## Slovenia

In curricular documents for teachers and students, mathematics is presented as one of the basic subjects of general *gymnasia* in which students learn mathematics concepts and structures, critical thinking, and reasoning; develop creativity, formal knowledge and skills; recognize the practical usefulness of mathematics; gain mathematics knowledge and competencies needed for future mathematics studies as well as learning in other subjects and everyday life. The gymnasium mathematics course is compulsory and the same for all future university students, regardless of their area of study. The national curriculum for advanced mathematics is available in the form of printed and e-books containing general goals, contents and topics, expected student outcomes, and recommendations for teaching, including the incorporation of ICT, homework, and assessments into mathematics courses. In addition to the curriculum that is written for teachers' use, the expected standards, list of topics and examples of questions for basic and advanced level of the mathematics *matura* examination exist in printed and e-documents for students.

Contents and topics are given in the general order of teaching the advanced mathematics course through four years. For each topic, expected goals for students are followed by list of specified topics to be taught, expected hours of lessons needed for the content, and didactical recommendations about use of ICT. Included also are suggestions and guidelines for connecting the topics with material from other academic areas and how the topics could be presented and taught in these contexts. There are some topics classified as optional or as left to the teacher's discretion based on the teacher's expectations for students' achievement. The prescribed topics in each compulsory and elective content are listed below.

Content Area	Topics
Sets and Logic	Basics of logic; sets
Numbers	Number sets with whole, rational, real, and complex numbers (mathematical induction and the polar form of complex numbers are optional topics)
Algebraic Expressions	Equations and inequalities and their methods of solution (parametric equations are optional); powers and roots
Geometry	Lines, angles, circles and triangles in a plane and in space; sines and cosines; the areas of 2-D geometric shapes and the volumes of 3-D shapes and sections; Cartesian coordinate systems; vectors in a plane and in space, scalar product (vector product is optional)
Functions	Limits, continuity, inverse and composite functions; linear functions; solving systems of linear equations; quadratic, exponential, rational, logarithmic and trigonometric functions; conic sections
Sequences and Series	
Calculus	Differential calculations; integrals; applications of integrals
Probability and Statistics	Combinatorics

Expected outcomes are given by main topics as a list of content and procedural knowledge, provided in the table below. Procedural knowledge outcomes include general skills and processes linked to mathematical knowledge but transferable also to other areas.

Knowledge	Expected Outcomes
Content Knowledge	<ul style="list-style-type: none"> <li>Calculate with numbers</li> <li>Use properties of sets</li> <li>Use logic in proofs</li> <li>Understand linear, power, root, quadratic, exponential, logarithmic, rational and trigonometric functions and calculate with them</li> <li>Draw graphs and use them in modeling</li> <li>Use Euclidean geometry and trigonometric functions in the context of Euclidean geometry; link Euclid geometry and vectors</li> <li>Use conic sections in problems</li> <li>Know and use arithmetic and geometric sequences and series, and apply them in financial mathematics and natural growth context</li> <li>Understand and use derivatives and determine tangents and simple extrema problems</li> <li>Know the meanings of indefinite and definite integrals; find indefinite integrals in simple situations, and use definite integrals for calculations of the area of a surface of revolution and volume of a solid of revolution</li> <li>Understand and use the fundamental principle of counting and other principles of combinatorics</li> <li>Know the classic definition of probability and calculate the probability of compound events</li> <li>Know statistical concepts, use them in other subject areas, and provide statistical analysis for a given problem</li> </ul>
Procedural Knowledge	<ul style="list-style-type: none"> <li>Abstract thinking</li> <li>Understanding of formal mathematical reasoning</li> <li>Analytical problem solving with different strategies</li> <li>Use of mathematics in everyday life (geometry, measurement, estimations, data analysis, interest expenses)</li> <li>Developing effective reading strategies for future learning</li> <li>Communicating mathematics in oral, written and other forms in the mother tongue and in one foreign language</li> <li>Designing and carrying out a research study and critically reporting findings</li> <li>Formulating research questions and hypotheses</li> <li>Thinking about necessary and sufficient conditions</li> <li>Using ICT and the Internet responsibly</li> <li>Making decisions and giving estimates of risks</li> </ul>

Cross-curricular connections are provided as examples of activities that can link together knowledge from different subjects and mathematics.

Didactic recommendations describe the compulsory use of ICT in as many possible forms and activities as possible:

- ◆ To develop skills
- ◆ To reach new knowledge
- ◆ To help students with disabilities
- ◆ To help with calculations, statistics and in communication

All available digital devices (computers, tablets, graphic classroom boards, advanced calculators) and specialized software for learning mathematics (geometry simulations, symbolic calculations, drawing) are encouraged to be used for learning mathematics.

Homework is presented as the basic form of self-motivated learning and primary source for discussions in a class. It is said to help student attain better knowledge and may indirectly influence students' grades. Students should be assessed by at least four written tests and one oral examination in class per year. Other forms are also suggested (projects, research, group work) with the recommendation that students be given enough opportunities to demonstrate their knowledge in different situations and are encouraged to develop responsibility for their own learning.

For the *matura*, students can decide whether to take the basic level or advanced level of the compulsory mathematics exam. The curriculum contents for both are the same, but required standards differ. The written test for the advanced level, in addition to compulsory items for the basic level, contains additional advanced level items. For oral examinations, the expected knowledge for the advanced level is specified in the *matura* standards (i.e. theoretical explanation of the definition of a limit versus the calculation of the limit only). Students receive grades from 1 to 5 for the *matura* exam at the basic level and from 1 to 8 at the advanced level. The sum of grades from all five *matura* subjects is used as a criterion for entrance to tertiary-level education programs with a limit on the number of new students.

## Sweden

Four consecutive mathematics courses, Mathematics 1–4, comprise the mathematics curriculum covered by Swedish advanced mathematics students in upper-secondary school. In addition, students can choose to take additional mathematics courses. All courses are defined by a national curriculum including the goal of the subject, core content, and assessment criteria. These curricula describe learning objectives in short texts and teachers are expected to interpret the brief descriptions.

The curriculum dictates that mathematics courses should give students the opportunity to develop their ability to:

- ◆ Use and describe the meaning of mathematical concepts and their inter-relationships
- ◆ Employ procedures and solve standard tasks with and without tools
- ◆ Formulate, analyze and solve mathematical problems, and assess selected strategies, methods and results
- ◆ Interpret a realistic situation and design a mathematical model, as well as use and assess a model's properties and limitations
- ◆ Follow, apply, and assess mathematical reasoning
- ◆ Communicate mathematical thinking orally, in writing, and in action
- ◆ Relate mathematics to its importance and use in other subjects, in a professional, social and historical context

These competencies are the same for all courses, but the core content differs.

Algebra is introduced in compulsory school, and given a more comprehensive coverage in upper-secondary school. Early on in upper-secondary school the concept of linear inequality as well as algebraic and graphical methods for solving linear equations and inequalities, and exponential equations are introduced. Students later learn about logarithms. Students learn to solve different kinds of equations, including exponential, second degree polynomial and root equations, as well as systems of linear equations. The core content covers the concept of absolute values, and the concepts of polynomial and rational expressions, and generalization of the laws of arithmetic for dealing with these concepts. Furthermore, the number system is extended through the introduction of the concept of complex numbers in connection with solving second-degree equations. Mathematics 4 gives a more comprehensive coverage of different aspects of complex numbers.

In Geometry, the core content is mostly found in the first two mathematics courses. In Mathematics 1, students are introduced to the concepts of sine, cosine and tangent, as well as vectors and their representations. Students add and subtract vectors and do scalar multiplication. Geometry is used in order to illustrate the concepts of definition, theorem and proof. Students

learn about the properties of the equation of a circle and are introduced to the unit circle in defining trigonometric concepts. In Mathematics 4, the core content contains a deeper coverage of trigonometry, for example methods for solving trigonometric equations.

Content relating to functions and calculus is found under the heading of Relationships and Change in all four mathematics courses. Students are taught about different kinds of functions and their properties. Calculus is added in Mathematics 3, starting with a brief introduction to continuous and discrete functions, as well as the concept of limits. Differentiation and use of the rules of differentiation for power and exponential functions, and also sums of functions, is described in the core content, as are algebraic and graphical methods for determining the value of the derivative of a function. Lessons should cover algebraic and graphical methods for solving extreme value problems using sign tables and second derivatives, and the relationship between the graph of a function and the first and second derivatives of a function. In Mathematics 4, the study of functions is expanded to include properties of trigonometric functions, logarithmic functions, compound functions and absolute values as functions. Lessons in differentiation and the use of the rules of differentiation for trigonometric, logarithmic, exponential and compound functions, and also the product and quotients of functions are included. In addition, students are expected to learn about algebraic and graphical methods for determining integrals with and without digital tools.

The core content also includes some arithmetic as well as probability and statistics, covered in the first two courses, and not as relevant to studies in advanced mathematics.

Problem solving is described as a core content in all four courses taken by advanced mathematics students in Sweden. Lessons cover strategies for mathematical problem solving including the use of digital media and tools, mathematical problems of importance in societal life and applications in other subjects, and mathematical problems related to the cultural history of mathematics.

## United States

The United States does not have a uniform curriculum for advanced mathematics. For TIMSS Advanced 2015, students were sampled from courses identified as calculus using the definitions from the School Codes for the Exchange of Data (SCED) course classification system. The SCED courses included two College Board Advanced Placement (AP) courses (AB and BC), two International Baccalaureate (IB) Diploma Programme courses (IB Mathematics Standard Level and IB Mathematics High Level), and other courses implemented at the state, district, or school level. Descriptions of courses and their content in school catalogues were reviewed to determine course eligibility. As a result, the students assessed in TIMSS Advanced 2015 participated in varying curricula. The AP and IB courses have specific curricula that are taught to all students regardless of the state, district or school in which they take them.

In AP Calculus AB, the curriculum is broken into three major topic areas: functions, graphs, and limits; derivatives; and integrals. Under functions, graphs, and limits, the curriculum covers analysis of graphs, limits of functions (including one-sided limits), asymptotic and unbounded behavior, and continuity as a property of functions. Under derivatives, the curriculum covers the concept of a derivative, derivative at a point, derivative as a function, second derivatives, and applications and computation of derivatives. Under integrals, the curriculum covers interpretations and properties of definite integrals, application of integrals, Fundamental Theorem of Calculus, techniques and application of antidifferentiation, and numerical approximation of definite integrals.

AP Calculus BC has a similar curriculum as AP Calculus AB, and covers all of the topics of AP Calculus AB, with additional material. Under functions, graphs, and limits, the AP Calculus BC curriculum additionally covers parametric, polar, and vector functions. AP Calculus BC also has a fourth major topic area: polynomial approximations and series. This topic covers the concept of series, series of constants, and Taylor series.

IB Mathematics Standard Level (SL) has a core curriculum that covers algebra, functions and equations, circular functions and trigonometry, matrices, vectors, statistics and probability, and calculus (differential and integral). The curriculum also requires all students to complete a portfolio of two individual pieces of work, based on mathematical investigation and mathematical modeling. IB Mathematics Higher Level (HL) has the same core curriculum and portfolio requirements as IB Mathematics SL, but additionally requires 40 hours of instruction in one of the following topics: statistics and probability, sets, relations and groups, series and differential equations, or discrete mathematics.

The other courses that students were sampled from are “Calculus and Analytic Geometry” and “Calculus”, with course curricula varying by state, district, or school.







# ADVANCED MATHEMATICS APPENDICES

TIMSS ADVANCED 2015 INTERNATIONAL RESULTS IN  
ADVANCED MATHEMATICS AND PHYSICS



**IEA**

**TIMSS & PIRLS**  
International Study Center  
Lynch School of Education, Boston College



**Exhibit MA.1: Countries Participating in the TIMSS Advanced 2015 Advanced Mathematics Assessment and in Earlier TIMSS Advanced Assessments**

Country	2015	2008	1995
France	●		●
Italy	●	●	●
Lebanon	●	●	
Norway	●	●	
Portugal	●		
Russian Federation	●	●	●
Slovenia	●	●	●
Sweden	●	●	●
United States	●		●

● Indicates participation in that testing cycle.

The Russian Federation participated in 2015 with an expanded population that included the more specialized students assessed in 1995 and 2008.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Appendix MB.1: Distribution of Items Included in the Advanced Mathematics Assessment by Content Domain, Cognitive Domain, and Item Format**

Advanced Mathematics Items	Multiple-Choice Items	Constructed Response Items	Total Items	Percentage of Score Points
<b>Content Domain</b>				
Algebra	19 (20)	18 (23)	37 (43)	35%
Calculus	21 (23)	13 (21)	34 (44)	36%
Geometry	19 (19)	12 (17)	31 (36)	29%
Total	59 (62)	43 (61)	102 (123)	100%
Percentage of Score Points	50%	50%		
<b>Cognitive Domain</b>				
Knowing	27 (29)	6 (7)	33 (36)	29%
Applying	22 (22)	18 (28)	40 (50)	41%
Reasoning	10 (11)	19 (26)	29 (37)	30%
Total	59 (62)	43 (61)	102 (123)	100%
Percentage of Score Points	50%	50%		

Score points are shown in parentheses.  
Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Appendix MC.1: Coverage of the TIMSS Advanced 2015 Target Population for Advanced Mathematics**

Country	International Target Population Coverage	Exclusions from National Target Population		
		School-Level Exclusions	Within-Sample Exclusions	Overall Exclusions
France	100%	4.6%	0.1%	4.7%
Italy	100%	0.5%	0.7%	1.1%
Lebanon	100%	1.3%	0.0%	1.3%
Norway	100%	1.4%	0.0%	1.4%
Portugal	100%	0.0%	0.3%	0.3%
Russian Federation	100%	0.2%	0.1%	0.3%
Russian Federation 6hr+	100%	1.0%	0.1%	1.1%
Slovenia	100%	0.3%	2.2%	2.5%
Sweden	100%	1.6%	0.1%	1.7%
United States	100%	0.0%	0.1%	0.1%

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

- 1 National Target Population does not include all of the International Target Population.
- 2 National Defined Population covers 90% to 95% of National Target Population.
- 3 National Defined population covers less than 90% of National Target population (but at least 77%).

**Exhibit MC.2: Size of the TIMSS Advanced 2015 Target Population for Advanced Mathematics, the Age Cohort, and the TIMSS Advanced Mathematics Coverage Index**

Country	Years of Formal Schooling*	Age Cohort Corresponding to the Final Year of Secondary School	Estimated Size of the Population of Students in the Final Year of Secondary School Taking the Advanced Mathematics Track or Program Targeted by TIMSS Advanced (Derived from TIMSS Advanced Student Sample)	Size of the Age Cohort Corresponding to the TIMSS Advanced Population Based on National Census Figures**	TIMSS Advanced Mathematics Coverage Index – the Percentage of the Entire Corresponding Age Cohort Covered by the TIMSS Advanced Target Population
France	12	18	172,178	801,889	21.5%
Italy	13	19	141,419	576,506	24.5%
Lebanon	12	18	4,457	113,204	3.9%
Norway	13	19	6,751	63,894	10.6%
Portugal	12	18	31,314	109,984	28.5%
Russian Federation	11	18	138,548	1,365,790	10.1%
Russian Federation 6hr+	11	18	25,830	1,365,790	1.9%
Slovenia	13	19	6,738	19,598	34.4%
Sweden	12	19	15,285	108,138	14.1%
United States	12	18	473,405	4,168,000	11.4%

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

\* Represents years of schooling counting from the first year of primary or basic education (first year of ISCED Level 1).

\*\* France: Estimate derived by dividing the population of 15–19-year-olds by 5 for the single year estimate. Data retrieved from INSEE (National Institute of Statistics and Economic Studies), Estimations de Population (résultats provisoires à fin 2015); [http://www.insee.fr/fr/themes/detail.asp?reg\\_id=99&ref\\_id=estim-pop](http://www.insee.fr/fr/themes/detail.asp?reg_id=99&ref_id=estim-pop).

Italy: Value is the total population of 19-year olds in Italy in 2015. Data retrieved from ISTAT (the National Statistics Institute); [http://dati.istat.it/Index.aspx?DataSetCode=DCIS\\_POPRES1](http://dati.istat.it/Index.aspx?DataSetCode=DCIS_POPRES1).

Lebanon: Value is the total population of 18-year olds in Lebanon in 2015. Data retrieved from <http://databank.worldbank.org/data/reports.aspx?source=health-nutrition-and-population-statistics;-population-estimates-and-projections&Type=TABLE&preview=on>.

Norway: Estimate derived by dividing the population of 15–19-year-olds by 5 for the single year estimate. Data retrieved from [https://stats.oecd.org/Index.aspx?DataSetCode=POP\\_PROJ](https://stats.oecd.org/Index.aspx?DataSetCode=POP_PROJ).

Portugal: Estimate derived by dividing the 2014 population of 15–19-year-olds by 5 for the single year estimate. Data retrieved from INE (Instituto Nacional de Estatística) Annual Estimates of Resident Population; <http://www.pordata.pt/en/Portugal/Resident+population+total+and+by+age+group-10>.

Russian Federation: Estimate derived by dividing the population of 15–19-year-olds by 5 for the single year estimate. Data retrieved from The Demographic Yearbook of Russia, 2015; [http://www.gks.ru/wps/wcm/connect/rosstat\\_main/rosstat/ru/statistics/publications/catalog/doc\\_1137674209312](http://www.gks.ru/wps/wcm/connect/rosstat_main/rosstat/ru/statistics/publications/catalog/doc_1137674209312).

Slovenia: Value is the total population of 18-year olds in Slovenia as of July 1st 2015. Data retrieved from the Statistical Office of the Republic of Slovenia; <http://pxweb.stat.si>.

Sweden: Value is the total population of 18-year olds as of December 31, 2014 (Born 1996). Data retrieved from Statistics Sweden; [http://www.statistikdatabasen.scb.se/pxweb/sv/ssd/START\\_\\_BE\\_\\_BE0101\\_\\_BE0101A/BefolkningR1860/table/tableViewLayout1/?rxid=06695d79-5fa1-41d1-81c1-3ae51dcd09b7](http://www.statistikdatabasen.scb.se/pxweb/sv/ssd/START__BE__BE0101__BE0101A/BefolkningR1860/table/tableViewLayout1/?rxid=06695d79-5fa1-41d1-81c1-3ae51dcd09b7).

United States: Value is the total population of 18-year olds as of July 1st 2015. Data retrieved from the US Census Annual Estimates of the Resident Population by Single Year of Age and Sex for the United States: April 1, 2010 to July 1, 2013; <https://www.census.gov/popest/data/national/asrh/2013/>. The post-census estimates are as of July 1 of each year. For the 18 year-olds estimate in 2015, the 2015 population was projected using the year to year changes from 2010 to 2013 and extending it to 2015.

The TIMSS Advanced Mathematics Coverage Index reflects the differences across countries in the proportion of the age cohort that are enrolled in these advanced courses in the final year of secondary education. In some countries, only a very select group of students was considered eligible for the study, while in others, a much larger group was included.

The TIMSS Advanced Mathematics Coverage Index (TAMCI) is defined as follows:

$$TAMCI = \frac{\text{Estimated total number of students in the advanced mathematics target population in 2015}}{\text{Total national population in the corresponding age cohort in 2015}} \times 100\%$$

The numerator is the total number of students eligible for TIMSS Advanced, estimated from the weighted sample data. These are students in the final year of secondary school taking the advanced mathematics track or program targeted by TIMSS Advanced, based on the TIMSS Advanced sample. The denominator is the size of the population age cohort corresponding to the average age of the students in the target populations and is based on national census figures.

**Appendix MC.3: School Sample Sizes - Advanced Mathematics**

Country	Number of Schools in Original Sample	Number of Eligible Schools in Original Sample	Number of Schools in Original Sample that Participated	Number of Replacement Schools that Participated	Total Number of Schools that Participated
France	146	145	144	0	144
Italy	120	120	104	9	113
Lebanon	355	354	251	0	251
Norway	136	134	133	0	133
Portugal	251	251	206	15	221
Russian Federation	346	346	346	0	346
Russian Federation 6hr+	181	163	163	0	163
Slovenia	80	77	69	0	69
Sweden	143	141	139	0	139
United States	348	316	230	11	241

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015



**Appendix MC.4: Student Sample Sizes - Advanced Mathematics**

Country	Within-School Student Participation (Weighted Percentage)	Number of Sampled Students in Participating Schools	Number of Students Withdrawn from Class/School	Number of Students Excluded	Number of Eligible Students	Number of Students Absent	Number of Students Assessed
France	96%	4,310	41	7	4,262	295	3,967
Italy	97%	3,547	28	30	3,489	171	3,318
Lebanon	98%	1,222	0	0	1,222	61	1,161
Norway	93%	2,756	31	1	2,724	187	2,537
Portugal	93%	4,581	109	15	4,457	389	4,068
Russian Federation	98%	7,758	2	12	7,744	186	7,558
Russian Federation 6hr+	98%	3,530	0	3	3,527	96	3,431
Slovenia	87%	3,360	1	42	3,317	395	2,922
Sweden	90%	4,450	85	2	4,363	426	3,937
United States	87%	3,488	57	2	3,429	475	2,954

Students attending a sampled class at the time the sample was chosen but leaving the class before the assessment was administered were classified as "withdrawn."

Students with a disability or language barrier that prevented them from participating in the assessment were classified as "excluded."

Students not present when the assessment was administered, and not subsequently assessed in a make-up session, were classified as "absent."

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Appendix MC.5: Participation Rates (Weighted) - Advanced Mathematics**

Country	School Participation		Class Participation	Student Participation	Overall Participation	
	Before Replacement	After Replacement			Before Replacement	After Replacement
France	99%	99%	100%	96%	95%	95%
Italy	88%	94%	99%	97%	85%	90%
‡ Lebanon	70%	70%	100%	98%	68%	68%
Norway	100%	100%	100%	93%	93%	93%
† Portugal	80%	87%	98%	93%	73%	80%
Russian Federation	100%	100%	100%	98%	98%	98%
Russian Federation 6hr+	100%	100%	100%	98%	98%	98%
Slovenia	89%	89%	96%	87%	75%	75%
Sweden	99%	99%	99%	90%	88%	88%
‡ United States	72%	76%	100%	87%	63%	66%

TIMSS Advanced guidelines for sampling participation: The minimum acceptable participation rates were 85% of both schools and students, or a combined rate (the product of school and student participation) of 75%. Participants not meeting these guidelines were annotated as follows:

† Met guidelines for sample participation rates only after replacement schools were included.

‡ Nearly satisfied guidelines for sample participation rates after replacement schools were included.

‡ Did not satisfy guidelines for sample participation rates.

SOURCE: IEA's Trends in International Mathematics and Science Study - TIMSS Advanced 2015

**Appendix MC.6: Trends in Student Populations – Advanced Mathematics**

Country	Years of Formal Schooling*			Average Age at Time of Testing			Overall Exclusion Rates**			Advanced Mathematics Coverage Index***			Overall Participation Rates		
	2015	2008	1995	2015	2008	1995	2015	2008	1995	2015	2008	1995	2015	2008	1995
France	12		12	18.0		18.2	4.7%		1.0%	21.5%		19.9%	95%		77%
Italy	13	13	13	18.9	19.0	19.1	1.1%	0.5%	3.8%	24.5%	19.7%	14.1%	90%	95%	68%
Lebanon	12	12		17.8	17.9		1.3%	1.3%		3.9%	5.9%		68%	83%	
Norway	13	13		18.7	18.8		1.4%	1.0%		10.6%	10.9%		93%	83%	
Russian Federation 6hr+	11	10/11	10	17.7	17.0	16.9	1.1%	0.0%	2.0%	1.9%	1.4%	2.0%	98%	98%	96%
Slovenia	13	12	12	18.8	18.8	18.9	2.5%	1.3%	6.0%	34.4%	40.5%	75.4%	75%	81%	42%
Sweden	12	12	12	18.8	18.8	18.9	1.7%	1.7%	0.2%	14.1%	12.8%	16.2%	88%	84%	89%
United States	12		12	18.1		18.0	0.1%		3.7%	11.4%		6.4%	66%		71%

\* Represents years of schooling counting from the first year of ISCED Level 1.

\*\* In 1995 exclusion rates for Advanced Mathematics were computed based on exclusion rates among all students in the final year of schooling. In the case of the Russian Federation, the figure presented in the 1995 International Report (43.0%) greatly overestimates the level of exclusions in the advanced mathematics population. The figure presented above (2.0%) includes two regions, North Ossetia and Chechen Republic, as well as non-Russian speaking students.

\*\*\* See Appendix MC.2 for a description of the Advanced Mathematics Coverage Index. The 1995 Advanced Mathematics Coverage Index for Italy was recomputed and is different than from the percentage reported in the 1995 International Report.

Russian Federation trend results are available only for the Intensive stream students (6hr+). The United States adjusted the 1995 sample to correspond with the course-taking definitions used in 2015, and the 1995 results were recomputed.

An empty cell indicates a country did not participate in that year's assessment.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Appendix MD.1: Average Percent Correct in the Advanced Mathematics Content and Cognitive Domains**

Country	Overall Advanced Mathematics	Advanced Mathematics Content Domains			Advanced Mathematics Cognitive Domains		
		Algebra	Calculus	Geometry	Knowing	Applying	Reasoning
France	36 (0.5)	41 (0.6)	34 (0.6)	32 (0.5)	49 (0.6)	31 (0.5)	30 (0.6)
Italy	31 (0.7)	33 (0.8)	32 (0.8)	29 (0.7)	42 (0.9)	29 (0.7)	25 (0.7)
Lebanon	50 (0.7)	52 (0.8)	51 (0.7)	47 (0.9)	64 (0.7)	45 (0.8)	44 (0.9)
Norway	37 (0.9)	37 (0.9)	35 (0.9)	38 (0.9)	46 (0.9)	33 (0.9)	33 (0.9)
Portugal	40 (0.5)	47 (0.6)	35 (0.5)	37 (0.5)	49 (0.5)	35 (0.5)	36 (0.7)
Russian Federation	43 (1.1)	48 (1.2)	36 (1.0)	45 (1.1)	52 (1.1)	41 (1.2)	37 (1.0)
Russian Federation 6hr+	54 (1.7)	61 (1.7)	46 (1.6)	57 (1.9)	63 (1.5)	52 (1.7)	50 (1.8)
Slovenia	37 (0.6)	43 (0.8)	30 (0.7)	37 (0.6)	49 (0.8)	34 (0.6)	28 (0.7)
Sweden	33 (0.6)	34 (0.7)	33 (0.6)	31 (0.6)	40 (0.7)	30 (0.6)	29 (0.6)
United States	43 (1.0)	45 (1.0)	46 (1.2)	37 (0.9)	54 (1.1)	39 (0.9)	38 (1.0)
International Avg.	39 (0.3)	42 (0.3)	37 (0.3)	37 (0.3)	49 (0.3)	35 (0.3)	33 (0.3)

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

# Appendix ME: Test–Curriculum Matching Analysis

TIMSS Advanced 2015 went to great lengths to ensure that comparisons of student achievement across countries would be as fair and equitable as possible. The [TIMSS Advanced 2015 Assessment Frameworks](#) were designed to specify the important aspects of advanced mathematics that participating countries agreed should be the focus of an international assessment of student achievement. The assessment items were developed through a collaborative process with national representatives to faithfully represent the specifications in the frameworks and were field tested extensively in participating countries. Finalizing the TIMSS Advanced 2015 advanced mathematics assessment involved a series of reviews by representatives of the participating countries, experts in mathematics, and testing specialists. At the end of this process, the National Research Coordinators (NRCs) from each country formally approved the TIMSS Advanced 2015 advanced mathematics assessment, thus accepting it as being sufficiently fair to compare their students’ advanced mathematics achievement with that of students from other countries.

Although the assessment was developed to represent agreed upon frameworks and was intended to have as much in common across countries as possible, it was unavoidable that the match between the advanced mathematics assessment (or test) and the advanced mathematics curriculum would not be the same in all countries. To restrict test items to just those topics included in the curricula of all participating countries and covered in the same sequence would severely limit test coverage and restrict the research questions that the study is designed to address. The test, therefore, inevitably has some items measuring topics unfamiliar to some students in some countries.

The Test-Curriculum Matching Analysis (TCMA) was conducted to investigate the extent to which the TIMSS Advanced 2015 advanced mathematics assessment matched each country’s curriculum. The TCMA also investigated the impact on a country’s performance of including only achievement items that were judged to be relevant to its own curriculum.<sup>1</sup>

To gather data about the extent to which the TIMSS Advanced 2015 advanced mathematics test matched the curricula of the participating countries, National Research Coordinators were asked to examine each achievement item and indicate whether the item was in their country’s intended curriculum for the advanced mathematics program(s) or track(s) assessed by TIMSS Advanced. Since an item might be in the curriculum for some but not all students in a country, coordinators were asked to consider an item included if it was in the intended curriculum for more than 50 percent of the students. All TIMSS Advanced 2015 participants took part in the TCMA analysis.

<sup>1</sup> Because there may also be curriculum areas covered in some countries that are not covered by the TIMSS Advanced 2015 tests, the TCMA does not provide complete information about how well the tests cover the curricula of the countries.

Exhibits ME.1 and ME.2 present the TCMA results for the TIMSS Advanced 2015 advanced mathematics test. Exhibit ME.1 shows the average percent correct on the advanced mathematics items judged appropriate by each country. Exhibit ME.2 shows the standard errors corresponding to the percentages presented in Exhibit ME.1.

In Exhibit ME.1, the bottom row of the exhibit shows the number of items, in terms of score points, identified as appropriate in each country. For advanced mathematics, the maximum number of score points in the assessment was 120 points.<sup>2</sup> Generally, the proportion of items judged appropriate was fairly high. From the bottom row, it can be seen that the United States and Slovenia judged almost all of the items (119 score points) to be appropriate as did Norway (118) and Italy (117). Lebanon (112), Portugal (111), Sweden (111), and France (109) judged over 90 percent of the items to be included in the curriculum, and the Russian Federation (91) judged over 75 percent to be included.

Because most countries indicated that at least some items were not included in their intended curriculum at the grade tested, the data were analyzed to determine whether the inclusion of these items had any effect on the international performance comparisons.<sup>3</sup>

The first data column of Exhibit ME.1 shows the average percent correct on all advanced mathematics test items for each country, together with its standard error. Subsequent columns show the performance of every country on those items judged appropriate by the country listed at the head of the column. Countries are presented in order of their performance based on average percent correct on all of the advanced mathematics items, from highest to lowest. To interpret this exhibit, choosing a country and reading across its row provides the average percent correct for the students in that country on the items selected by each of the countries listed along the top of the exhibit. For example, Lebanon, where the average percent correct was 51 percent on the set of advanced mathematics items that it judged appropriate, had, on average, 49 percent correct on the items judged appropriate by the Russian Federation, 50 percent on the items selected by the United States, 51 percent on the items selected by Portugal, and so forth.

The column for a country listed at the top shows how each of the other countries performed on the set of items selected as appropriate for that country's students. Using the set of advanced mathematics items selected by Portugal as an example, 51 percent of these items, on average, were answered correctly by students in Lebanon, 44 percent by students in the Russian Federation, 43 percent by students in the United States, and so forth. The shaded diagonal element in the exhibit shows how each country performed on the set of items that it selected based on its own curriculum. Thus, students from Portugal averaged 42 percent correct on the set of items identified by Portugal for the analysis.

For each country's selected items, the international averages across the participating countries are presented in a row in the lower part of the exhibit for each subject. These show that the

2 The TIMSS Advanced 2015 advanced mathematics assessment contained 102 items yielding 123 score points. However, following item review, the 102 items and 123 score points in the advanced mathematics assessment were reduced to 101 items and 120 score points.

3 It should be noted that the advanced mathematics achievement presented in Exhibits ME.1 is based on average percent correct (the percentage of students in a country answering each item correctly, averaged across all items), which is different from the average scale scores that are presented in main tables of the report.

**Exhibit ME.1: Average Percent Correct for the Test-Curriculum Matching Analyses in Advanced Mathematics**

Based on a subset of items specifically identified by each country as addressing its curriculum

Read across the row to compare that country's performance based on the test items included by each of the countries across the top. Read down the column under a country name to compare the performance of the country down the left on the items included by the country listed on the top. Read along the diagonal to compare performance for each different country based on its own decisions about the test items to include.

Country	Average Percent Correct on All Items	Lebanon	Russian Federation	United States	Portugal	Norway	Slovenia	France	Sweden	Italy
	Lebanon	50 (0.7)	51	49	50	51	50	50	50	51
Russian Federation	43 (1.1)	44	44	43	44	43	43	43	42	43
United States	43 (1.0)	43	41	43	43	43	43	42	42	43
Portugal	40 (0.5)	39	40	40	42	40	39	40	40	40
Norway	37 (0.9)	37	37	37	37	37	37	37	36	36
Slovenia	37 (0.6)	37	37	37	37	36	37	37	36	36
France	36 (0.5)	36	36	36	36	36	36	36	35	36
Sweden	33 (0.6)	33	33	33	33	33	33	32	32	33
Italy	31 (0.7)	32	30	32	32	32	31	31	31	32
International Avg.	39 (0.3)	39	38	39	39	39	39	39	38	39
Number of Items (Score Points) Identified*	120	112	91	119	111	118	119	109	111	117

\* Of the 102 items in the Advanced Mathematics test, some extended-response items were scored on a two-point scale, resulting in 123 score points. Following item review, one item was deleted and the point value of two items were reduced, resulting in 101 items and 120 score points.  
 (.) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

selections of items by the participating countries varied only slightly in average difficulty, which is not surprising given that countries included most items in the advanced mathematics assessment.

Comparing the diagonal element for a country with the overall average percent correct shows the difference between performance on the set of items chosen as appropriate for that country and performance on the test as a whole. Countries generally performed similarly or a little better on their own items when compared to their performance on all items. To illustrate, the average percent correct for the Russian Federation across all the advanced mathematics items was 43 percent. The diagonal element shows that students from the Russian Federation had a slightly greater average percent correct (44 percent) across the set of items selected as appropriate for Russian students than they did overall. Portugal had the biggest difference between the two measures with Portuguese students achieving 42 percent correct on the items judged to be in their curriculum while achieving 40 percent correct on all items.

It is clear that the selection of items did not have a major effect on the relative performance in advanced mathematics among TIMSS Advanced 2015 countries. Countries that had relatively high or low performance across all of the items in the assessment also had relatively high or low

**Exhibit ME.2: Standard Errors for the Test-Curriculum Matching Analyses in Advanced Mathematics**

Based on a subset of items specifically identified by each country as addressing its curriculum

Read across the row to compare that country's performance based on the test items included by each of the countries across the top. Read down the column under a country name to compare the performance of the country down the left on the items included by the country listed on the top. Read along the diagonal to compare performance for each different country based on its own decisions about the test items to include.

Country	Average Percent Correct on All Items	Lebanon	Russian Federation	United States	Portugal	Norway	Slovenia	France	Sweden	Italy
		Lebanon	50 (0.7)	0.7	0.8	0.7	0.7	0.7	0.7	0.7
Russian Federation	43 (1.1)	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
United States	43 (1.0)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Portugal	40 (0.5)	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Norway	37 (0.9)	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.9	0.8
Slovenia	37 (0.6)	0.6	0.7	0.6	0.7	0.6	0.6	0.6	0.6	0.6
France	36 (0.5)	0.5	0.6	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Sweden	33 (0.6)	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
Italy	31 (0.7)	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
International Avg.	39 (0.3)	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
Number of Items (Score Points) Identified*	120	112	91	119	111	118	119	109	111	117

\* Of the 102 items in the Advanced Mathematics test, some extended-response items were scored on a two-point scale, resulting in 123 score points. Following item review, one item was deleted and the point value of two items were reduced, resulting in 101 items and 120 score points.  
 ( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

performance on each of the various sets of items selected for the TCMA. For example, Lebanon had the highest average percent correct not only on the assessment as a whole, but also on all of the different item selections, with the Russian Federation, the United States, and Portugal next in order (with some ties) on practically all selections of items.<sup>4</sup>

The TCMA results provide evidence that the TIMSS Advanced 2015 advanced mathematics assessment constitutes a reasonable basis for comparing the achievement of the participating countries. This result is not unexpected, since making the assessment as fair as possible was a major consideration in test development. The fact that all countries indicated that most items were appropriate for their students means that the different average percent correct estimates were based on many of the same items. Insofar as countries rejected items that would be difficult for their students, these items do not greatly affect the overall pattern of relative performance.

4 Small differences in performance between adjacent countries shown in this exhibit usually are not statistically significant. The standard errors for the average percent correct statistics based on the TIMSS Advanced 2015 sample are provided in Exhibit ME.2. For any sample average shown in Exhibit ME.1, it can be said with 95 percent confidence that the corresponding value in the population falls between the sample estimate plus or minus 2 standard errors.



**Appendix MF.1: Percentiles of Achievement in Advanced Mathematics**

Country	5th Percentile	10th Percentile	25th Percentile	50th Percentile	75th Percentile	90th Percentile	95th Percentile
France	348 (5.8)	371 (5.4)	413 (3.1)	462 (3.3)	511 (3.5)	555 (4.1)	581 (5.4)
Italy	238 (9.1)	273 (9.0)	342 (9.1)	428 (6.4)	501 (4.6)	561 (5.8)	594 (7.9)
Lebanon	419 (5.8)	444 (5.6)	485 (3.9)	532 (3.8)	577 (5.1)	619 (3.6)	645 (7.5)
Norway	340 (8.1)	369 (5.3)	411 (5.6)	459 (5.0)	508 (5.2)	550 (6.4)	578 (6.3)
Portugal	363 (5.5)	389 (3.8)	434 (3.7)	482 (3.5)	532 (3.4)	577 (3.2)	601 (4.8)
Russian Federation	298 (10.2)	335 (8.4)	406 (7.7)	489 (6.9)	564 (6.0)	625 (7.0)	662 (9.4)
Russian Federation 6hr+	358 (17.3)	405 (16.8)	476 (11.2)	546 (8.4)	610 (7.7)	665 (7.5)	696 (11.6)
Slovenia	322 (7.0)	353 (6.8)	403 (4.6)	459 (4.0)	515 (4.7)	569 (5.2)	599 (5.9)
Sweden	267 (10.3)	305 (7.1)	365 (5.5)	433 (5.3)	501 (4.3)	555 (4.5)	584 (5.6)
United States	315 (12.6)	352 (10.5)	419 (8.3)	491 (5.3)	554 (5.8)	608 (4.8)	640 (8.2)

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.  
Note: Percentiles are defined in terms of percentages of students at or below a point on the scale.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

**Appendix MF.2: Standard Deviations of Achievement in Advanced Mathematics**

Country	Overall		Females		Males	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
France	463 (3.1)	71 (1.5)	449 (3.1)	66 (2.1)	475 (3.4)	74 (1.5)
Italy	422 (5.3)	110 (3.2)	427 (6.1)	107 (4.1)	419 (6.6)	111 (3.9)
Lebanon	532 (3.1)	69 (1.8)	533 (4.8)	67 (3.3)	531 (3.9)	71 (2.4)
Norway	459 (4.6)	72 (2.0)	453 (5.1)	70 (2.3)	463 (5.2)	72 (2.4)
Portugal	482 (2.5)	73 (1.4)	481 (3.0)	70 (1.4)	483 (3.1)	76 (2.0)
Russian Federation	485 (5.7)	111 (3.4)	480 (6.0)	109 (3.9)	489 (6.2)	113 (3.8)
Russian Federation 6hr+	540 (7.8)	101 (5.3)	530 (9.0)	101 (6.0)	549 (7.5)	101 (5.1)
Slovenia	460 (3.4)	84 (2.6)	449 (3.5)	81 (3.0)	476 (4.9)	85 (3.1)
Sweden	431 (4.0)	97 (2.2)	424 (5.1)	95 (3.6)	436 (4.6)	98 (2.3)
United States	485 (5.2)	98 (3.6)	470 (5.3)	94 (3.1)	500 (6.4)	101 (5.9)

( ) Standard errors appear in parentheses. Because of rounding some results may appear inconsistent.

SOURCE: IEA's Trends in International Mathematics and Science Study – TIMSS Advanced 2015

# Appendix MG: Organizations and Individuals Responsible for TIMSS Advanced 2015

## Introduction

TIMSS Advanced 2015 was a collaborative effort involving hundreds of individuals around the world. This appendix acknowledges the individuals and organizations for their contributions. Given that work on TIMSS Advanced 2015 has spanned approximately four years and has involved so many people and organizations, this list may not include all who contributed. Any omission is inadvertent. TIMSS Advanced 2015 also acknowledges the students, teachers, and school principals who contributed their time and effort to the study. This report would not be possible without them.

## Management and Coordination

TIMSS Advanced was conducted by IEA's TIMSS & PIRLS International Study Center at Boston College, which has responsibility for the direction and management of the TIMSS and PIRLS projects, including design, development, and implementation. Headed by Executive Directors Drs. Ina V.S. Mullis and Michael O. Martin, the study center is located in the Lynch School of Education. In carrying out the project, the TIMSS & PIRLS International Study Center worked closely with the IEA Secretariat in Amsterdam, which managed country participation, was responsible for verification of all translations produced by the participating countries, and coordinated the school visits by International Quality Control Monitors. Staff at the IEA Data Processing and Research Center in Hamburg worked closely with participating countries to organize sampling and data collection operations and to check all data for accuracy and consistency within and across countries; Statistics Canada in Ottawa was responsible for school and student sampling activities; and Educational Testing Service in Princeton, New Jersey consulted on psychometric methodology, provided software for scaling the achievement data, and replicated the achievement scaling for quality assurance.

The Project Management Team, comprising the study directors and representatives from the TIMSS & PIRLS International Study Center, IEA Secretariat and IEA Data Processing and Research Center, Statistics Canada, and ETS met twice a year throughout the study to discuss the study's progress, procedures, and schedule. In addition, the study directors met with members of IEA's Technical Executive Group twice yearly to review technical issues.

To work with the international team and coordinate within-country activities, each participating country designates an individual to be the TIMSS National Research Coordinator (NRC). The NRCs have the challenging task of implementing TIMSS in their countries in accordance with the TIMSS guidelines and procedures. In addition, the NRCs provide feedback and contributions throughout the development of the TIMSS assessment. The quality of the TIMSS assessment and data depends on the work of the NRCs and their colleagues in carrying out the complex sampling, data collection, and scoring tasks involved. Continuing the tradition of exemplary work established in previous cycles of TIMSS, the TIMSS Advanced 2015 NRCs performed their many tasks with dedication, competence, energy, and goodwill, and have been commended by the IEA Secretariat, the TIMSS & PIRLS International Study Center, the IEA Data Processing and Research Center, and Statistics Canada for their commitment to the project and the high quality of their work.

## Funding

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Michael O. Martin, *Executive Director*

Pierre Foy, *Director of Sampling, Psychometrics, and Data Analysis*

Paul Connolly, *Director, Graphic Design and Publications*

Ieva Johansone, *Associate Research Director, Operations and Quality Control*

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